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Higgs boson production in association with three jets

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Abstract: The scattering amplitudes for Higgs + 5 partons are computed, with the Higgs boson produced via gluon fusion in the large $M_{\rm top}$ limit. A parton-level analysis of Higgs + 3 jet production via gluon fusion and via weak-boson fusion is presented, and the effectiveness of a central-jet veto is analysed.

Keywords: Higgs Physics, QCD.

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Contents

1. Introduction

The mechanism that governs the electroweak symmetry breaking is at present the largest unknown in the Standard Model (SM) of elementary particle physics. The canonical mechanism, the Higgs model, is a keystone of the SM and its supersymmetric extensions. However, it is based on the existence of a CP-even scalar particle, the Higgs boson, which has not been detected yet and is the most wanted particle of the Fermilab Tevatron and the CERN Large Hadron Collider (LHC) physics programmes. The direct search in the $e^+e^- \to ZH$ process at the CERN LEP2 collider has posed a lower bound of 114.1 GeV on the SM Higgs mass, M_H [1]–[5]. On the other hand, a SM Higgs boson with a mass of the order of 200 GeV or less is favoured by analyses of the electroweak precision data [6]. Thus the intermediate mass region ($M_H \lesssim 200\,\mathrm{GeV}$) seems to be preferred.

In pp-collisions, the dominant production mechanism over the entire Higgs mass range accessible at LHC is via gluon fusion $gg \to H$ [7], where the coupling of the Higgs to the gluons is mediated by a heavy quark loop, (left-hand panel of figure 1). In the SM, the leading contribution comes from the top quark, the contributions from other quarks being at least smaller by a factor $\mathcal{O}(M_{\mathrm{b}}^2/M_{\mathrm{top}}^2)$. The second largest production mechanism is via weak-boson fusion (WBF), $qq \to qqH$ (right-hand panel of figure 1).[1] In addition, it is a key component of the program to measure the Higgs boson couplings at LHC [9].

[1] In terms of significance, i.e. ratio of signal over root of background $S/\sqrt{B}$, Higgs production via WBF may be larger than the one via gluon fusion [8].

Higgs production via WBF occurs as the scattering between two (anti)quarks with weak-boson (W or Z) exchange in the t-channel and with the Higgs boson radiated off the weak-boson propagator. Since the distribution functions of the incoming valence quarks peak at values of the momentum fractions $x \approx 0.1$ to 0.2, Higgs production via WBF tends to produce naturally two highly energetic outgoing quarks. In addition, the weak-boson mass provides a natural cutoff on the weak-boson propagator, thus two jets with a transverse energy typically of the order of a fraction of the weak-boson mass can be easily produced. Therefore a large fraction of the events of the total cross section for Higgs production via WBF occurs with two jets with a large rapidity interval between them, typically one at forward and the other at backward rapidities. By considering Higgs production in association with two jets, and by selecting events with a large dijet invariant mass and with a large rapidity separation between the jets, the WBF production channel can be made larger than the gluon fusion one [10, 11].

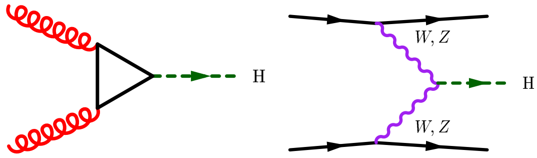

Figure 1: Higgs production via gluon fusion (left) and via WBF (right).

In refs. [10, 11], Higgs + 2 jet events have been analysed at leading order in α_S. A broader dijet mass distribution, together with a tendency to have larger rapidity intervals between the two jets, for WBF than for gluon fusion was found. However, Higgs + 2 jet production via gluon fusion features a strong dependence on the renormalisation scale, because it is a calculation to $\mathcal{O}(\alpha_S^4)$. The next-to-leading order (NLO) corrections to Higgs + 2 jet production via WBF are known and have been found to be typically modest, of the order of 5 to 10% [12]. Thus Higgs production via WBF is under a good theoretical control. NLO corrections to Higgs + 2 jet production via gluon fusion are not known. Such a calculation would be quite formidable. In fact, since the Higgs boson is produced via a loop, a calculation of the production rate is quite involved, even at leading order in α_S: in Higgs + 2 jet production via gluon fusion up to pentagon loops occur. Evaluating the NLO corrections would require to calculate amplitudes with up to either hexagon loops or pentagon double loops. On the other hand, it is known that the NLO corrections to the total rate of Higgs production via gluon fusion are large, increasing the production rate by up to 80% [13, 14], and so are the NLO corrections to Higgs + 1 jet production (computed in the large $M_{\rm top}$ limit) [15, 16, 17, 18]. Thus, we are led to expect that also the unknown NLO corrections to Higgs + 2 jet production be large.

Even though a full NLO calculation of Higgs + 2 jet production via gluon fusion is not feasible with the present technology, it could be done in the large top-quark mass limit, $M_H^2 \ll 4M_{\rm top}^2$, whereby the gluon-Higgs coupling via the top-quark loop is given by a low-energy theorem through an effective lagrangian [19, 20],

$$\mathcal{L}_{\rm eff} = \frac{1}{4} A H G^a_{\mu\nu} G^{a\ \mu\nu}\,, \tag{1.1}$$

with $G^a_{\mu\nu}$ the field strength of the gluon field and H the Higgs-boson field (the effective coupling A is given by $A = \alpha_S/(3\pi v)$, where v is the vacuum expectation value parameter,

$v^2 = (G_F\sqrt{2})^{-1} = (246\ \mathrm{GeV})^2$). For Higgs production in association with jets, one must require in addition that the jets and Higgs transverse energies are smaller than the top-quark mass [21, 22].

In this work, we present the tree-level scattering amplitudes for the process Higgs $\to$ 5 partons, with the Higgs boson coupling to gluons through the effective vertex of the large $M_{\rm top}$ limit. The amplitudes have been computed at fixed parton helicities. These amplitudes can be used to compute Higgs + 3 jet production via gluon fusion, which we will analyse in section 3; they contribute to the radiative corrections of the inclusive production of Higgs + 2 jets to NLO; they can also be used to complete the calculation of the collinear splitting functions of a gluon into four partons (only the splitting functions of a gluon into four gluons are known [23]). The splitting functions of a parton into four partons are some of the building blocks of a recently proposed method to compute the Altarelli-Parisi kernels to next-to-next-to-leading order accuracy [24].

The paper is organised as follows: in the Introduction we have described the basic features of Higgs + 2 jet production via WBF and via gluon fusion, which motivate our choice of the final-state topology in Higgs + 3 jet production; in section 2 we introduce the scattering amplitudes for Higgs + 5 partons, whose explicit expressions are collected in appendix B; in section 3 we present Higgs + 3 jet production via gluon fusion and via WBF; in section 4 we present our conclusions.

2. Amplitudes for Higgs + 5 partons

The scattering amplitudes for Higgs + 5 partons, with the Higgs boson coupling to gluons through the effective vertex of the large $M_{\rm top}$ limit, have been computed at fixed parton helicities, and have been decomposed into colour-stripped gauge-invariant sub-amplitudes, whose explicit expression is reported in appendix B. Here we show the colour decomposition of the amplitudes.

Since the Higgs boson is a colour singlet, the colour structure of the QCD amplitudes for Higgs + n partons is the same as the one of the tree n-parton amplitudes [25]. We repeat it here for later convenience.

The colour decomposition of the tree n-gluon amplitudes is [25, 23, 26][2]

$$\begin{aligned}\mathcal{M}_n(1,\ldots,n) &= 2^{(n-2)/2} g^{n-2} \sum_{\sigma\in S_n/\mathbb{Z}_n} \mathrm{tr}(T^{a_{\sigma_1}}\cdots T^{a_{\sigma_n}}) m_n(\sigma_1,\ldots,\sigma_n)\\ &= 2^{(n-2)/2}\frac{(ig)^{n-2}}{2}\times\\ &\quad\times\sum_{\sigma\in S_{n-2}} f^{a_1 a_{\sigma_2} x_1} f^{x_1 a_{\sigma_3} x_2}\cdots f^{x_{n-3} a_{\sigma_{(n-1)}} a_n} m_n(1,\sigma_2,\ldots,\sigma_{n-1},n)\,, \qquad (2.1)\end{aligned}$$

where $S_n/\mathbb{Z}_n$ are the non-cyclic permutations of the n gluons, and A is given below eq. (1.1). The dependence on the particle helicities and momenta in the sub-amplitude, and on the

[2] We use the standard normalisation of the fundamental representation matrices, $\mathrm{tr}(T^a T^b) = \delta^{ab}/2$. This is the origin of the factor $2^{n/2}$ which we make explicit rather than carrying it over in the sub-amplitudes, as in ref. [23]. The additional factor 2 is due to the effective coupling of the Higgs to the gluons. For $n = 4$, eq. (2.1) reproduces the normalisation of the sub-amplitudes of ref. [27].

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gluon colours in the trace, is implicit in labelling each leg with the index i. Helicities and momenta are defined as if all particles were outgoing. The gauge invariant sub-amplitudes m satisfy the relations [28],

$$\begin{aligned} & m(1,2,\ldots,n-1,n) = m(n,1,2,\ldots,n-1) && \text{cyclicity} \\ & m(1,2,\ldots,n) = (-1)^n m(n,\ldots,2,1) && \text{reflection} \\ & m(1,2,3,\ldots,n) + m(2,1,\ldots,n) + \ldots + m(2,3,\ldots,1,n) = 0 && \text{dual Ward identity}\,. \end{aligned} \tag{2.2}$$

The colour decomposition of the tree amplitudes for $(n-2)$ gluons and a $q\bar{q}$ pair is

$$\mathcal{M}_n(q,\bar{q};3,\ldots,n) = 2^{(n-4)/2} g^{n-2} \sum_{\sigma\in S_{(n-2)}} (T^{a_{\sigma_3}} \ldots T^{a_{\sigma_n}})_i^{\;\bar{\jmath}}\, m_n(1_q, 2_{\bar{q}}; \sigma_3, \ldots, \sigma_n)\,, \tag{2.3}$$

where $S_{(n-2)}$ is the permutation group of the $(n-2)$ gluons. Reflection symmetry is like in eq. (2.2), for gluons and/or quarks alike.

The colour decomposition of the tree amplitudes for two $q\bar{q}$ pairs and $(n-4)$ gluons is

$$\begin{aligned} \mathcal{M}_n(1_q, 2_{\bar{q}}, 3_Q, 4_{\overline{Q}}; 5, \ldots, n) = {} & 2^{(n-6)/2} g^{n-2} \times \\ & \times \sum_{k=0}^{n-4} \sum_{\sigma\in S_k} \sum_{\rho\in S_l} \Big[(T^{a_{\sigma_1}} \ldots T^{a_{\sigma_k}})_{i_3}^{\;\bar{\imath}_2}\, (T^{a_{\rho_1}} \ldots T^{a_{\rho_l}})_{i_1}^{\;\bar{\imath}_4} \times \\ & \qquad \times m_n(1_q, 2_{\bar{q}}, 3_Q, 4_{\overline{Q}}; \sigma_1, \ldots, \sigma_k; \rho_1, \ldots, \rho_l) + \\ & \qquad + \frac{1}{N_c} (T^{a_{\sigma_1}} \ldots T^{a_{\sigma_k}})_{i_1}^{\;\bar{\imath}_2}\, (T^{a_{\rho_1}} \ldots T^{a_{\rho_l}})_{i_3}^{\;\bar{\imath}_4} \times \\ & \qquad \times \tilde{m}_n(1_q, 2_{\bar{q}}, 3_Q, 4_{\overline{Q}}; \sigma_1, \ldots, \sigma_k; \rho_1, \ldots, \rho_l) \Big]\,, \end{aligned} \tag{2.4}$$

with $k+l=n-4$. The sums are over the partitions of $(n-4)$ gluons between the two quark lines, and over the permutations of the gluons within each partition. For $k=0$ or $l=0$, the colour strings reduce to Kronecker delta's.

Several checks on the calculation were performed. At the level of colour ordered amplitudes we verified the gauge invariance, the generic $1\to 3$ collinear limits, the $g\to gggg$ collinear limit [23] and the $m_H, p_H \to 0$ limit where the amplitudes reduce to the pure QCD five-parton amplitudes. At the level of squared amplitudes summed over colour and helicity we compared numerically with Alpha [29, 30] and MadGraph [31, 32] and found agreement at the machine level precision.

3. Higgs + 3 jet production

A distinguishing feature of WBF is that to leading order no colour is exchanged in the t-channel [33, 34]. To $\mathcal{O}(\alpha_S)$, gluon radiation occurs only as bremsstrahlung off the quark legs (since no colour is exchanged in the t-channel in the Born process, no gluon exchange is possible to $\mathcal{O}(\alpha_S)$, except for a tiny contribution due to equal-flavour quark scattering with $t \leftrightarrow u$ channel exchange).

The different gluon radiation pattern expected for Higgs production via WBF versus its major backgrounds, namely $t\bar{t}$ production and QCD $WW+2$ jet production, is at the core of the central-jet veto proposal, both for heavy [35] and light [36] Higgs searches. A veto of any additional jet activity in the central rapidity region is expected to suppress more the backgrounds than the signal, because the QCD backgrounds are characterised by quark or gluon exchange in the t-channel. The exchanged partons, being coloured, are expected to radiate off more gluons.

For the analysis of the Higgs coupling to gauge bosons, Higgs + 2 jet production via gluon fusion may also be treated as a background. When the two jets are separated by a large rapidity interval, the scattering process is dominated by gluon exchange in the t-channel. Therefore, like for the QCD backgrounds, the bremsstrahlung radiation is expected to occur everywhere in rapidity.[3] In order to analyse that, we consider Higgs + 3 jet production via WBF and via gluon fusion, at LHC.[4] On the three jets, we require the set of cuts

$$p_j^{\mathrm{T}} > 20\,\mathrm{GeV}\,, \qquad |y_j| < 5\,, \qquad R_{jj} > 0.6\,, \tag{3.1}$$

where p_j^{T} is the transverse momentum of a final state parton, y_j is its rapidity and R_{jj} is the separation of the two partons in rapidity versus azimuthal angle plane

$$R_{jj} = \sqrt{\Delta y_{jj}^2 + \phi_{jj}^2}\,. \tag{3.2}$$

In addition, like in ref. [12], we select the two jets with the highest transverse energy as the *tagging jets* of our events, and require them to pass the cuts

$$|y_{j1} - y_{j2}| > 4.2\,, \qquad y_{j1} \cdot y_{j2} < 0\,, \qquad m_{jj} > 600\,\mathrm{GeV}\,, \tag{3.3}$$

i.e. the two tagging jets must be well separated, must be in opposite detector hemispheres and must possess a large dijet invariant mass. In figure 2, we plot the distribution in the rapidity of the third jet, measured with respect to the rapidity average of the tagging jets, $y_{\mathrm{rel}} = y_{j3} - (y_{j1} + y_{j2})/2$, for gluon fusion and for WBF. Here and in figure 3 the Higgs mass was set to $M_H = 120\,\mathrm{GeV}$, the CTEQ5L set of parton distribution functions [38] was used along with the corresponding one-loop running of α_S. The factorisation scales were set equal to the geometric average of the p^{T} of the jets. About the choice of the renormalisation scale, note that Higgs + 3 jet production via gluon fusion is an $\mathcal{O}(\alpha_S^5)$ process, and a leading order process. Thus, its dependence on the renormalisation scale is very large. Since Higgs + 3 jet production with two tagging jets that pass the cuts (3.3) is dominated by configurations with gluon exchange in the t channel, a set of "natural" scale choices is given by the replacement $\alpha_S^5 \to \alpha_S^2(M_H)\alpha_S(p_1^{\mathrm{T}})\alpha_S(p_2^{\mathrm{T}})\alpha_S(p_3^{\mathrm{T}})$, in analogy with Higgs + 2 jet production with two forward jets [11, 22]. However, in order to reduce the dependence on the scale choices, in figure 2 we normalise the distribution of the third jet to the Higgs + 3 jet production rate. The different dynamical generation of the third jet is apparent: in WBF its production in the central rapidity region is naturally suppressed.

[3] An analogous difference in the gluon radiation pattern is expected in $Z + 2$ jet production via WBF fusion versus QCD production of $Z + 2$ jets [37].

[4] WBF results have been obtained with MadEvent [32].

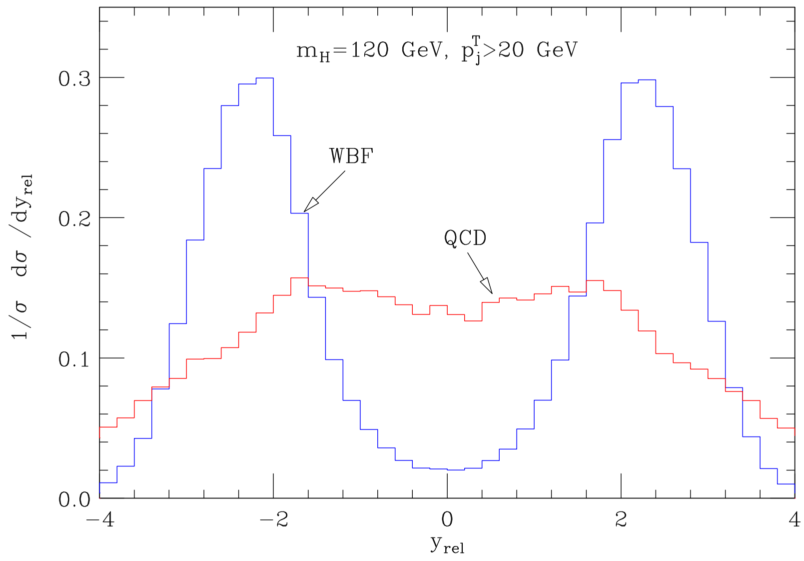

Figure 2: The distribution in the rapidity of the third jet, measured with respect to the rapidity average of the tagging jets, $y_{rel} = y_{j3} - (y_{j1} + y_{j2})/2$.

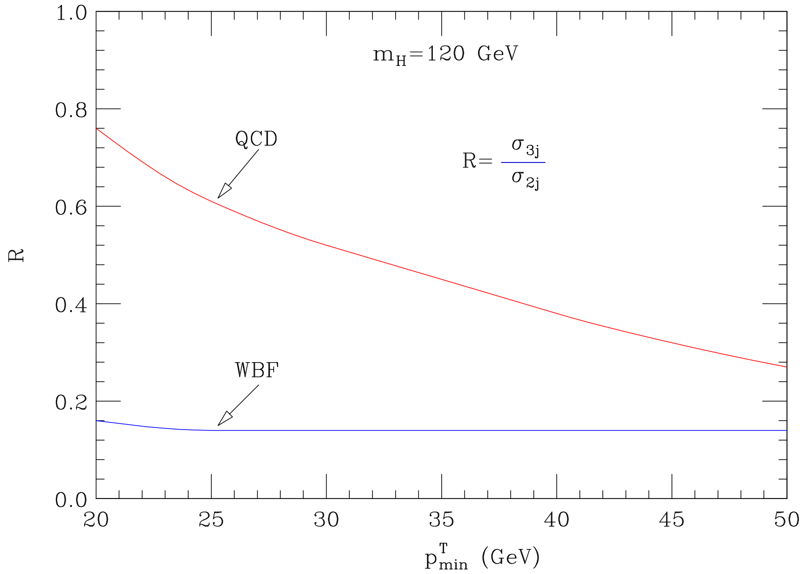

Figure 3: The ratio of the Higgs + 3 jet production rate to Higgs + 2 jet, as a function of $p^T_{\rm min}$.

The central-jet veto discards all events with three or more jets if any jet other than the tagging jets is produced above a given cut $p^T_{\rm min}$. Our jet selection implies then that the third largest jet in p^T be produced above $p^T_{\rm min}$. For the central-jet veto proposal it is of interest to analyse the ratio R of the Higgs + 3 jet production rate to Higgs + 2 jet: the larger is R, the more effective the central-jet veto is expected to be. In figure 3, we analyse R as a function of $p^T_{\rm min}$ for gluon fusion and for WBF. Both Higgs + 3 jet and Higgs + 2 jet production were evaluated to leading order in α_S.[5] We note

[5]For consistency with Higgs + 3 jet, also the tagging jets of Higgs + 2 jet production have been taken

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that the gluon fusion process is monotonically dependent on $p^{\mathrm{T}}_{\min}$ while WBF is basically independent of it. Thus the effectiveness of the central-jet veto grows as $p^{\mathrm{T}}_{\min}$ decreases.

4. Conclusions

In this work we have computed the scattering amplitudes for Higgs + 5 partons, with the Higgs boson produced via gluon fusion in the large M_{top} limit. Using those amplitudes, we have performed a parton-level analysis of Higgs + 3 jet production, via gluon fusion and via WBF. Since our ultimate goal is to collect information useful to study the Higgs coupling to gauge bosons, Higgs production via gluon fusion is treated as a background that must be distinguished from WBF and contained as much as possible. To that effect, in the literature forward jet tagging for Higgs + 2 jet production has been proposed, augmented by a central-jet veto for the production of a Higgs in association with three or more jets. We have analysed the jet veto proposal by comparing Higgs + 3 jet production via gluon fusion and via WBF. We have found that the more severe the cut on the third jet, the more effective the jet veto.

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A. Helicity amplitudes

We use helicity amplitudes, defined in terms of massless Dirac spinors $\psi_\pm(p)$ of fixed helicity,

$$\psi_\pm(p) = \frac{1\pm\gamma_5}{2}\psi(p) \equiv |p^\pm\rangle\,, \qquad \overline{\psi_\pm(p)} \equiv \langle p^\pm|\,, \tag{A.1}$$

spinor products,

$$\langle pk\rangle \equiv \langle p^-|k^+\rangle\,, \qquad [pk] \equiv \langle p^+|k^-\rangle\,, \qquad \text{with } \langle pk\rangle^* = \mathrm{sign}(p^0k^0)\,[kp]\,, \tag{A.2}$$

currents,

$$\begin{aligned}\langle i^-|p_{k_1}\dots p_{k_{2n+1}}|j^-\rangle &\equiv \langle i^-|\not{p}_{k_1}\dots\not{p}_{k_{2n+1}}|j^-\rangle\\ \langle i^-|p_{k_1}\dots p_{k_{2n}}|j^+\rangle &\equiv \langle i^-|\not{p}_{k_1}\dots\not{p}_{k_{2n}}|j^+\rangle\,.\end{aligned} \tag{A.3}$$

Using the decomposition of the fermion propagator

$$\not{p} = |p^+\rangle\langle p^+| + |p^-\rangle\langle p^-|\,, \tag{A.4}$$

above the cut $p^{\mathrm{T}}_{\min}$.

the simplest current can be written as

$$\langle i^- | p_k | j^- \rangle = \langle ik\rangle\, [kj] \tag{A.5}$$

and every other current can be written in terms of products of spinor products by applying eq. (A.4) repeatedly on it.

In order to set up the sign and symmetry conventions of our amplitudes, we collect the known sub-amplitudes for Higgs + three and four partons.

A.1 Sub-amplitudes for Higgs + three partons

The colour decomposition of the tree amplitudes for Higgs plus three gluons is given in eq. (2.1), with $n = 3$. The independent sub-amplitudes are [39]

$$\begin{aligned} m_3(1^+, 2^+, 3^+) &= iA\frac{M_H^4}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}\,, \\ m_3(1^-, 2^+, 3^+) &= iA\frac{[23]^3}{[12][13]}\,. \end{aligned} \tag{A.6}$$

All of the other sub-amplitudes can be obtained by relabelling and by use of reflection symmetry, eq. (2.2), and parity inversion. Parity inversion flips the helicities of all particles, and it is accomplished by the substitution $\langle i\, j\rangle \leftrightarrow [j\, i]$.

The colour decomposition of the tree amplitudes for Higgs, a gluon and a $q\bar{q}$ pair is given in eq. (2.3) for $n = 3$. There is only one independent sub-amplitude [39]

$$m_3(1_q^-, 2_{\bar{q}}^+; 3^+) = iA\frac{[23]^2}{[12]}\,. \tag{A.7}$$

All other sub-amplitudes can be obtained by use of parity inversion and charge conjugation.[6] Charge conjugation swaps quarks and antiquarks without inverting helicities.

A.2 Sub-amplitudes for Higgs + four partons

The colour decomposition of the amplitudes for Higgs plus four gluons is given in eq. (2.1) for $n = 4$. The independent sub-amplitudes are [27, 39]

$$\begin{aligned} m_4(1^+, 2^+, 3^+, 4^+) &= iA\frac{M_H^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}\,, \\ m_4(1^-, 2^+, 3^+, 4^+) &= iA\left(\frac{\langle 1^-|p_H|3^-\rangle^2[24]^2}{t_{124}s_{12}s_{14}} + \frac{\langle 1^-|p_H|4^-\rangle^2[23]^2}{t_{123}s_{12}s_{23}} + \frac{\langle 1^-|p_H|2^-\rangle^2[34]^2}{t_{134}s_{14}s_{34}}\right) - \\ &\quad -iA\frac{[24]}{[12]\langle 23\rangle\langle 34\rangle[41]}\left\{s_{23}\frac{\langle 1^-|p_H|2^-\rangle}{\langle 41\rangle} + s_{34}\frac{\langle 1^-|p_H|4^-\rangle}{\langle 12\rangle} - [24]t_{234}\right\}, \\ m_4(1^-, 2^-, 3^+, 4^+) &= iA\left(\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} + \frac{[34]^4}{[12][23][34][41]}\right), \end{aligned} \tag{A.8}$$

[6]In performing parity inversion, there is a factor of -1 for each pair of quarks participating in the amplitude.

with the Higgs momentum in a current defined through momentum conservation $p_H = -\sum_{i=1}^{n} p_i$. All of the other sub-amplitudes can be obtained by relabelling and by use of reflection symmetry, eq. (2.2), and parity inversion.

The colour decomposition of the amplitudes for Higgs, two gluons and a $q\bar{q}$ pair is given in eq. (2.3), with $n = 4$. The independent sub-amplitudes are [39],[7]

$$\begin{aligned} m_4(1_q^+, 2_{\bar{q}}^-; 3^+, 4^+) &= iA\left[\frac{\langle 2^-|p_H|4^-\rangle^2}{t_{123}}\frac{[13]}{\langle 23\rangle}\left(\frac{1}{s_{12}}+\frac{1}{s_{13}}\right) - \frac{\langle 2^-|p_H|3^-\rangle^2}{t_{124}s_{12}}\frac{[14]}{\langle 24\rangle} - \right. \\ &\qquad\left. - \frac{\langle 2^-|p_H|1^-\rangle^2}{[12]\langle 23\rangle\langle 24\rangle\langle 34\rangle}\right] \\ m_4(1_q^+, 2_{\bar{q}}^-; 3^-, 4^+) &= -iA\left(\frac{\langle 23\rangle^3}{\langle 12\rangle\langle 24\rangle\langle 34\rangle} - \frac{[14]^3}{[12][13][34]}\right), \\ m_4(1_q^+, 2_{\bar{q}}^-; 3^+, 4^-) &= -iA\left(-\frac{[13]^2[23]}{[12][24][34]} + \frac{\langle 14\rangle\langle 24\rangle^2}{\langle 12\rangle\langle 13\rangle\langle 34\rangle}\right). \end{aligned} \tag{A.9}$$

All of the other sub-amplitudes can be obtained by relabelling and by use of parity inversion, reflection symmetry and charge conjugation.

The colour decomposition of the amplitudes for Higgs and two $q\bar{q}$ pairs is given in eq. (2.4), with $n = 4$, which can be re-cast as,

$$\mathcal{M}_4(1_q, 2_{\bar{q}}; 3_Q, 4_{\overline{Q}}) = g^2\frac{1}{2}\left(\delta_{i_3}^{\bar{\imath}_2}\delta_{i_1}^{\bar{\imath}_4} - \frac{1}{N_c}\delta_{i_1}^{\bar{\imath}_2}\delta_{i_3}^{\bar{\imath}_4}\right) m_4(1_q, 2_{\bar{q}}; 3_Q, 4_{\overline{Q}}). \tag{A.10}$$

There is one independent sub-amplitude [39],

$$m_4(1_q^+, 2_{\bar{q}}^-; 3_Q^+, 4_{\overline{Q}}^-) = -iA\left(\frac{\langle 24\rangle^2}{\langle 12\rangle\langle 34\rangle} + \frac{[13]^2}{[12][34]}\right). \tag{A.11}$$

All the other sub-amplitudes, except for $m_4(1_q^+, 2_{\bar{q}}^-; 3_Q^-, 4_{\overline{Q}}^+)$ can be obtained by relabelling and by use of parity inversion, reflection symmetry and charge conjugation. $m_4(1_q^+, 2_{\bar{q}}^-; 3_Q^-, 4_{\overline{Q}}^+)$ is obtained from eq. (A.11) by inverting the parity on the current $\langle 3^-|\gamma^\mu|4^-\rangle$. That amounts to swap the labels 3 and 4 in eq. (A.11).

B. Sub-amplitudes for Higgs + five partons

In appendices B.1, B.2 and B.3 we collect the colour-stripped sub-amplitudes for Higgs + five partons. The squared amplitude for Higgs + five gluons, summed over colours, is worked out in section B.4.

B.1 Sub-amplitudes for Higgs + five gluons

The colour decomposition of the amplitudes for Higgs plus five gluons is given in eq. (2.1) for $n = 5$. There are three independent sub-amplitudes, we may take to be the ones with

[7] For the colour ordering on the fermion line we choose the convention of ref. [25], which is the opposite of the one used in ref. [39].

zero, one and two gluons of negative helicity. They are

$$m_5(1^+,2^+,3^+,4^+,5^+) = iA\frac{M_H^4}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,4\rangle\langle 4\,5\rangle\langle 5\,1\rangle}\,, \tag{B.1}$$

$$\begin{aligned}
&m_5(1^-,2^+,3^+,4^+,5^+) = \\
&= iA\Big\{\frac{\langle 1^-|(p_2+p_3)p_H|1^+\rangle^2}{\langle 2\,3\rangle\langle 4\,5\rangle}\times \\
&\quad\times\left[\frac{[2\,5]}{s_{12}s_{15}\langle 1\,3\rangle\langle 1\,4\rangle}+\frac{[3\,2]}{s_{12}\langle 1\,4\rangle\langle 1\,5\rangle t_{123}}-\frac{[4\,5]}{s_{15}\langle 1\,2\rangle\langle 1\,3\rangle t_{145}}\right]+ \\
&\quad+\frac{\langle 1^-|(p_2+p_5)p_H|1^+\rangle^2[2\,5]^2}{s_{12}s_{15}\langle 1\,3\rangle\langle 1\,4\rangle\langle 3\,4\rangle t_{125}}-\frac{\langle 1^-|p_H|2^-\rangle^2}{s_{15}\langle 1\,3\rangle\langle 3\,4\rangle\langle 4\,5\rangle[1\,2]}\times \\
&\quad\times\left[[2\,5]+\frac{\langle 3\,4\rangle[1\,2][4\,5]\langle 1^-|p_4+p_5|3^-\rangle}{t_{145}s_{2H}}\right]- \\
&\quad-\frac{[2\,5]t_{234}\langle 1^-|p_H|5^-\rangle^2}{s_{12}\langle 1\,4\rangle\langle 3\,4\rangle\langle 2\,3\rangle[1\,5]s_{5H}}+\frac{[2\,3]\langle 1^-|p_H|5^-\rangle^2\langle 1^-|p_2+p_3|4^-\rangle}{s_{12}\langle 1\,4\rangle\langle 2\,3\rangle t_{123}s_{5H}}+ \\
&\quad+\frac{\langle 1^-|p_H|5^-\rangle^3}{\langle 1\,2\rangle\langle 1\,4\rangle\langle 2\,3\rangle\langle 3\,4\rangle[1\,5]s_{5H}}- \\
&\quad-\frac{\langle 1^-|p_H|4^-\rangle^2\langle 1^-|p_2+p_3|5^-\rangle}{s_{12}\langle 2\,3\rangle s_{4H}}\left[\frac{[5\,2]}{s_{15}\langle 1\,3\rangle}+\frac{[2\,3]}{\langle 1\,5\rangle t_{123}}\right] \\
&\quad-\frac{[2\,5]^2\langle 1^-|p_2+p_5|3^-\rangle\langle 1^-|p_H|4^-\rangle^2}{s_{12}s_{15}\langle 1\,3\rangle t_{125}s_{4H}}+\frac{\langle 1^-|p_H|3^-\rangle^2}{\langle 1\,2\rangle s_{15}s_{3H}}\times \\
&\quad\times\left[\frac{\langle 1^-|p_4+p_5|2^-\rangle}{\langle 4\,5\rangle}\left(\frac{[2\,5]}{[1\,2]\langle 1\,4\rangle}+\frac{[4\,5]}{t_{145}}\right)+\frac{[2\,5]^2\langle 1^-|p_2+p_5|4^-\rangle}{[2\,1]\langle 1\,4\rangle t_{125}}\right]- \\
&\quad-\frac{\langle 1^-|p_H|2^-\rangle^2\langle 1^-|p_3+p_4|5^-\rangle}{s_{15}\langle 1\,3\rangle\langle 3\,4\rangle\langle 4\,5\rangle s_{2H}}\Big\}\,,
\end{aligned} \tag{B.2}$$

$$\begin{aligned}
&m_5(1^-,2^-,3^+,4^+,5^+) = \\
&= iA\Big\{\frac{-\langle 1\,2\rangle^2}{\langle 1\,3\rangle\langle 3\,4\rangle\langle 4\,5\rangle[1\,2]s_{15}}[[1\,2]\langle 1^-|p_3+p_4|5^-\rangle-[2\,5]t_{345}]+ \\
&\quad+\frac{1}{4\langle 1\,3\rangle\langle 1\,4\rangle\langle 4\,5\rangle[1\,2][2\,5]s_{15}s_{23}t_{123}}\times \\
&\quad\times\Big\{2[3\,5]\Big[2\langle 1\,2\rangle^2[2\,5][3\,5]\Big(-s_{23}\langle 1\,5\rangle\langle 3\,5\rangle[2\,5]+\langle 1\,3\rangle[1\,2][3\,4]\times \\
&\qquad\qquad\times(\langle 1\,5\rangle\langle 3\,4\rangle+\langle 1\,4\rangle\langle 3\,5\rangle)\Big)- \\
&\qquad-2\langle 1\,3\rangle\langle 2\,3\rangle[2\,5](s_{13}+s_{23})\langle 1^-|p_4+p_5|3^-\rangle^2+ \\
&\qquad+\langle 1\,2\rangle\Big(-2\langle 1\,5\rangle^2\langle 2\,3\rangle[2\,5]^2[3\,5]s_{23}- \\
&\qquad\quad-2\langle 1\,3\rangle^2[2\,5][3\,4][3\,5]\langle 5^-|p_2p_1-p_1p_2|4^+\rangle+ \\
&\qquad\quad+\langle 1\,3\rangle\langle 2\,3\rangle\times \\
&\qquad\quad\times\Big(2\langle 1\,5\rangle[2\,5][3\,5]\langle 3^+|-p_2p_5-p_5p_1|2^-\rangle+ \\
&\qquad\qquad+\langle 1\,4\rangle^2[1\,2][3\,4]([2\,5][3\,4]+[2\,4][3\,5])+ \\
&\qquad\qquad+\langle 1\,4\rangle\langle 1\,5\rangle[1\,2][3\,5](7[2\,5][3\,4]+[2\,4][3\,5])\Big)\Big)\Big]+ \\
&\quad+\langle 1\,2\rangle[4\,5]\Big[\langle 1\,2\rangle[2\,5]s_{12}\Big(\langle 1\,4\rangle[3\,4]\langle 4^-|2p_1+p_5|2^-\rangle+
\end{aligned}$$

$$
\begin{aligned}
&\qquad\qquad + \langle 1\,5\rangle[3\,5]\langle 4^-|2p_1+2p_3+p_5|2^-\rangle\Big) + \\
&\quad + \langle 1^-|p_4+p_5|3^-\rangle\Big(-4s_{23}^2\langle 1\,4\rangle[2\,5]+\langle 1\,3\rangle^2[1\,2][3\,5]\times \\
&\qquad\qquad \times\langle 4^-|-2p_1-p_5|3^-\rangle - \\
&\qquad\qquad - 2s_{23}\langle 1\,3\rangle(-2\langle 1\,4\rangle[1\,5][2\,3]+\langle 4\,5\rangle[2\,5][3\,5])\Big)+ \\
&\quad + 2\langle 1\,2\rangle[2\,3]\Big(\langle 1\,3\rangle[1\,2]\times \\
&\qquad\qquad \times(\langle 1\,4\rangle^2[1\,5][3\,4]+ \\
&\qquad\qquad\quad + \langle 1\,5\rangle[3\,5]\langle 4^-|-p_1-p_2|5^-\rangle)+\langle 2\,3\rangle[2\,5]\times \\
&\qquad\qquad \times(\langle 1\,4\rangle[3\,4]\langle 4^-|-3p_1-p_5|2^-\rangle+ \\
&\qquad\qquad\quad + \langle 1\,5\rangle[3\,5]\langle 4^-|-2p_1-p_5|2^-\rangle)\Big)\Big] \\
&\quad - \langle 1\,2\rangle^2\langle 1\,4\rangle[2\,3][4\,5]^2\times \\
&\quad \times(2s_{23}\langle 4^-|p_1+p_5|2^-\rangle+s_{12}\langle 4^-|2p_1+3p_5|2^-\rangle+ \\
&\qquad + \langle 1\,3\rangle[1\,2]\langle 4^-|-2p_1-p_5|3^-\rangle)\Big\} + \\
&+\frac{[4\,5]}{\langle 1\,3\rangle\langle 4\,5\rangle[2\,5]s_{15}s_{23}t_{145}}\times \\
&\times\Big[-\langle 1\,3\rangle\langle 2\,3\rangle[3\,5]\langle 1^-|p_4+p_5|3^-\rangle^2 \\
&\quad + \langle 1\,2\rangle[3\,5]\Big(-\langle 1\,3\rangle\langle 1\,4\rangle[3\,4]s_{23}-\langle 1\,5\rangle^2\langle 2\,3\rangle[2\,5][3\,5]- \\
&\qquad\qquad - \langle 1\,5\rangle\langle 1\,3\rangle[3\,5](s_{23}+s_{25})\Big) - \\
&\quad - \langle 1\,2\rangle^2[2\,5](-s_{23}\langle 1\,4\rangle[3\,4]-\langle 1\,5\rangle[3\,5](s_{23}+s_{35}))- \\
&\quad - \langle 1\,2\rangle^3[2\,3][2\,5]t_{145}\Big] - \frac{\langle 1\,2\rangle\langle 1^-|p_3+p_4|5^-\rangle}{\langle 1\,3\rangle\langle 1\,4\rangle\langle 3\,4\rangle[1\,2]s_{15}t_{125}}\times \\
&\times[-(s_{12}+s_{15})\langle 1^-|p_3+p_4|5^-\rangle+\langle 5^+|p_2(p_3+p_4)p_5|1^+\rangle- \\
&\quad - \langle 1\,2\rangle[2\,5]t_{345}] + \frac{2\langle 1\,2\rangle}{\langle 1\,3\rangle\langle 3\,4\rangle\langle 4\,5\rangle[1\,2][2\,5]s_{15}s_{23}}\times \\
&\times[\langle 3\,4\rangle[1\,2][4\,5](2\langle 1\,2\rangle^2[2\,3][2\,5]+\langle 1\,3\rangle[3\,5]\langle 1^-|p_4+p_5|3^-\rangle+ \\
&\quad + \langle 1\,2\rangle(\langle 1\,5\rangle[2\,5][3\,5]+\langle 1\,4\rangle([2\,5][3\,4]-[2\,3][4\,5])))- \\
&\quad - 2[2\,5]s_{23}t_{345}\langle 1^-|p_2-p_3-p_4|5^-\rangle] - \\
&-\frac{1}{4\langle 1\,3\rangle\langle 1\,4\rangle\langle 3\,4\rangle[1\,2][1\,5][2\,5]s_{23}s_{5H}}\times \\
&\times\Big\{4\langle 1\,3\rangle^2[3\,5]^3s_{23}\langle 3^-|p_1+p_2|5^-\rangle+[3\,5][4\,5]\times \\
&\quad \times\Big[\langle 1\,2\rangle^2\langle 3\,4\rangle[2\,5]^2(2s_{23}-s_{12})-2\langle 1\,3\rangle[3\,5]s_{23}\times \\
&\qquad \times(5\langle 1\,4\rangle\langle 2\,3\rangle[2\,5]-\langle 1\,3\rangle\langle 4^-|6p_1+p_2|5^-\rangle) \\
&\qquad + s_{12}\langle 1\,3\rangle^2\langle 3\,4\rangle[3\,5]^2\Big]+\langle 1\,4\rangle[3\,5][4\,5]^2\times \\
&\quad \times\Big[-\langle 1\,2\rangle\langle 3\,4\rangle[2\,5]s_{12}-6s_{23}\times \\
&\qquad \times(\langle 1\,4\rangle\langle 2\,3\rangle[2\,5]+\langle 1\,3\rangle\langle 4^-|-2p_1-p_2|5^-\rangle)+ \\
&\qquad + s_{12}\langle 1\,3\rangle\langle 3\,4\rangle[3\,5]\Big]+4\langle 1\,4\rangle^2[4\,5]^3s_{23}\langle 4^-|p_1+p_2|5^-\rangle\Big\} -
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1}{4\langle 1\,3\rangle\langle 1\,4\rangle[1\,2][2\,5]s_{15}s_{23}s_{5H}t_{123}}\times\\
&\times\Big\{\langle 1\,2\rangle^3[2\,3][2\,5]^2[4\,5]s_{12}s_{45}-4\langle 1\,5\rangle\langle 2\,3\rangle[2\,5][3\,4][3\,5]\times\\
&\quad\times(s_{13}+s_{23})\langle 1^-|p_3+p_4|5^-\rangle^2+\langle 1\,2\rangle^3[2\,5][4\,5]\times\\
&\quad\times\Big(2s_{45}[2\,3]\langle 2^+|p_3p_2+2p_1p_3|5^-\rangle+\\
&\qquad+\langle 1\,4\rangle[1\,2][2\,5]\langle 4^+|4p_3p_5+p_5p_2|3^-\rangle+\\
&\qquad+\langle 1\,5\rangle[1\,2](-\langle 2\,4\rangle[2\,5]^2[3\,4]+\\
&\qquad\qquad+[3\,5](-s_{34}[2\,5]+\langle 5^+|-p_1p_4+p_4p_1|2^-\rangle))\Big)+\\
&+\langle 1\,2\rangle^2\Big[2\langle 1\,3\rangle[1\,2][3\,5]\times\\
&\qquad\times\Big(2\langle 1\,4\rangle[2\,5][3\,5][4\,5]\langle 5^-|p_2-p_3|4^-\rangle+\langle 1\,5\rangle(2\langle 2\,3\rangle[2\,4][2\,5][3\,5]^2+\\
&\qquad\quad+[4\,5](-2s_{34}[2\,5][3\,5]+\langle 1\,4\rangle[1\,5][2\,3][4\,5]+\\
&\qquad\qquad+[2\,5]\langle 5^+|-p_2p_4+2p_4p_2|3^-\rangle))\Big)+\\
&\qquad+[4\,5]\Big(2\langle 1\,4\rangle[2\,5]^2s_{23}\langle 4^+|-p_5p_2-2p_3p_5|3^-\rangle+\\
&\qquad\quad+\langle 1\,5\rangle(-2\langle 2\,3\rangle[2\,5](\langle 2\,4\rangle[2\,3][2\,5]^2[3\,4]+\\
&\qquad\qquad+\langle 1\,4\rangle[3\,5]([1\,4][2\,3][2\,5]-\\
&\qquad\qquad\quad-3[1\,2][2\,4][3\,5]))+\\
&\qquad\qquad+\langle 1\,4\rangle[1\,2][4\,5](-3s_{34}[2\,5][3\,5]+\langle 2\,4\rangle[2\,5]\times\\
&\qquad\qquad\quad\times(-[2\,5][3\,4]+3[2\,3][4\,5])+\\
&\qquad\qquad\quad+\langle 1\,4\rangle(-[1\,4][2\,5][3\,5]+\\
&\qquad\qquad\qquad+2[4\,5]([1\,5][2\,3]+[1\,3][2\,5])))))\Big)\Big]+\\
&+\langle 1\,2\rangle\Big[2\langle 1\,4\rangle\langle 1\,5\rangle\langle 2\,3\rangle[4\,5](-s_{23}[2\,5][3\,5]([2\,5][3\,4]+2[2\,4][3\,5])+\\
&\qquad\qquad+[4\,5](-s_{34}[2\,3][2\,5][3\,5]+[2\,3][2\,5]\langle 5^+|-p_2p_4+p_4p_2|3^-\rangle+\\
&\qquad\qquad\quad+\langle 1\,4\rangle(-[1\,4][2\,3][2\,5][3\,5]+[1\,2][3\,5]\times\\
&\qquad\qquad\quad\times([2\,5][3\,4]+[2\,4][3\,5])+2[1\,5][2\,3]^2[4\,5])))+\\
&\qquad+\langle 1\,3\rangle^2[3\,5]^2(4s_{25}\langle 1\,4\rangle[1\,2][3\,4][4\,5]+\\
&\qquad\quad+\langle 1\,5\rangle(4\langle 2\,3\rangle[3\,5]([1\,5][2\,3][2\,4]+[1\,2]([2\,5][3\,4]+[2\,4][3\,5]))+\\
&\qquad\qquad+[1\,2][4\,5](-s_{34}[3\,5]+\langle 5^+|p_1p_4-p_4p_1|3^-\rangle+\\
&\qquad\qquad\quad+\langle 5^+|-3p_2p_4+p_4p_2|3^-\rangle)))+\\
&\qquad+\langle 1\,3\rangle[3\,5](-4[1\,4][2\,5][3\,4][4\,5]s_{23}s_{25}+\\
&\qquad\quad+\langle 1\,5\rangle(-4s_{23}\langle 2\,3\rangle[2\,4][2\,5][3\,5]^2+\\
&\qquad\qquad+\langle 1\,4\rangle[1\,2][4\,5]^2(-s_{34}[3\,5]+\langle 5^+|p_1p_4-p_4p_1|3^-\rangle+\\
&\qquad\qquad\quad+\langle 5^+|-3p_2p_4+p_4p_2|3^-\rangle)+\\
&\qquad\qquad+2\langle 2\,3\rangle[4\,5]([2\,3][2\,5]\langle 3^+|-p_4p_3-p_2p_4|5^-\rangle+\\
&\qquad\qquad\quad+\langle 1\,4\rangle([3\,5](-[1\,4][2\,3][2\,5]+\\
&\qquad\qquad\qquad+[1\,2](5[2\,5][3\,4]+3[2\,4][3\,5]))+\\
&\qquad\qquad\qquad+2[1\,5][2\,3]([2\,4][3\,5]+[2\,3][4\,5])))))\Big]\Big\}-
\end{aligned}
$$

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$$
-\frac{\langle 1^-|p_H|5^-\rangle}{2\langle 1\,3\rangle\langle 1\,4\rangle\langle 3\,4\rangle[1\,5][2\,5]s_{23}s_{5H}t_{234}}\times
$$

$$
\times\Big[\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,4\rangle[3\,5]^2(\langle 1\,3\rangle[3\,4][2\,5]+\langle 1\,4\rangle[2\,4][4\,5])
$$

$$
-2\langle 2\,3\rangle[3\,5]\langle 1^-|p_3+p_4|5^-\rangle^2(s_{23}+s_{34})+
$$

$$
+\langle 1\,2\rangle\langle 1\,3\rangle\langle 3\,4\rangle[3\,5]^2\langle 4^+|-p_2p_3-p_3p_4|5^-\rangle+
$$

$$
+\langle 1\,2\rangle^2\langle 3\,4\rangle[2\,5][3\,5][4\,5](s_{23}+s_{24})+
$$

$$
+2\langle 1\,3\rangle\langle 2\,4\rangle[2\,3][4\,5](\langle 1\,3\rangle\langle 2\,3\rangle[3\,5]^2+\langle 1\,4\rangle\langle 2\,4\rangle[4\,5]^2)+
$$

$$
+\langle 1\,2\rangle\langle 2\,4\rangle\langle 3\,4\rangle[3\,5][4\,5]^2\langle 1^-|p_4-p_3|2^-\rangle+
$$

$$
+\langle 1\,4\rangle[3\,5][4\,5]^2(-4s_{23}\langle 1\,3\rangle\langle 2\,4\rangle-s_{34}\langle 1\,2\rangle\langle 3\,4\rangle)\Big]+
$$

$$
+\frac{\langle 1\,2\rangle[3\,5][4\,5]}{4\langle 1\,3\rangle[1\,2][2\,5]s_{15}s_{23}s_{4H}}\langle 1^-|p_H|4^-\rangle[(s_{12}-2s_{23})[2\,5]+\langle 1\,3\rangle[1\,2][3\,5]]-
$$

$$
-\frac{1}{4\langle 1\,3\rangle\langle 1\,4\rangle[1\,2][2\,5]s_{15}s_{23}s_{4H}t_{123}}\times
$$

$$
\times\Big\{4\langle 1\,4\rangle\langle 2\,3\rangle[2\,5][3\,5]^2(s_{13}+s_{23})\langle 1^-|p_3+p_5|4^-\rangle^2-2\langle 1\,2\rangle^3[2\,5][4\,5]s_{12}\times
$$

$$
\times(-2s_{45}[2\,3][2\,5]+\langle 1\,4\rangle[1\,2](-[2\,5][3\,4]+[2\,3][4\,5]))+
$$

$$
+\langle 1\,2\rangle^3\Big[4[2\,5](\langle 1\,3\rangle[1\,2][3\,5](2s_{34}[2\,5][3\,4]-s_{45}[2\,3][4\,5])+
$$

$$
+[2\,5][4\,5](s_{23}s_{45}[2\,3]-\langle 1\,5\rangle[1\,2]\langle 3^+|p_4p_3+p_2p_4|5^-\rangle))-
$$

$$
-\langle 1\,4\rangle[1\,2](2\langle 2\,3\rangle[2\,5](-2[2\,5]^2[3\,4]^2-[2\,3]^2[4\,5]^2+
$$

$$
+[2\,5][3\,4](2[2\,4][3\,5]+[2\,3][4\,5]))+
$$

$$
+[4\,5](2\langle 1\,3\rangle(2[1\,5][2\,3]([2\,5][3\,4]+[2\,4][3\,5])+
$$

$$
+[3\,5](-2[1\,4][2\,3][2\,5]-[1\,3][2\,4][2\,5]+
$$

$$
+[1\,2]([2\,5][3\,4]+[2\,4][3\,5])))+
$$

$$
+[2\,5]([2\,4][3\,5](3s_{35}-s_{25})+
$$

$$
+\langle 1\,5\rangle(-2[1\,2][3\,5][4\,5]+[1\,5]([2\,5][3\,4]-3[2\,3][4\,5])))))\Big]+
$$

$$
+\langle 1\,2\rangle[3\,5]\Big[2\langle 1\,3\rangle^3[1\,2][3\,4][3\,5]\langle 5^+|2p_2p_4+p_4p_1|3^-\rangle+2\langle 1\,4\rangle\langle 1\,5\rangle\langle 2\,3\rangle[2\,5][4\,5]\times
$$

$$
\times(2[2\,3]\langle 5^+|-p_2p_3+p_3p_2|4^-\rangle+([2\,3](s_{35}-s_{25})
$$

$$
+\langle 1\,5\rangle([1\,5][2\,3]-2[1\,2][3\,5]))[4\,5])+
$$

$$
+\langle 1\,3\rangle^2[3\,5](-4\langle 1\,5\rangle\langle 2\,4\rangle[1\,2][2\,5][3\,4][4\,5]+
$$

$$
+\langle 1\,4\rangle([1\,2][4\,5]([3\,4](3s_{25}-s_{35})+\langle 1\,5\rangle([1\,5][3\,4]-2[1\,3][4\,5]))-
$$

$$
-2\langle 2\,3\rangle[3\,4](2[1\,5][2\,3][2\,4]+[1\,2](5[2\,5][3\,4]+[2\,4][3\,5]))))-
$$

$$
-\langle 1\,3\rangle(-4\langle 1\,5\rangle\langle 2\,4\rangle[2\,5]^2[4\,5][3\,4]s_{23}+
$$

$$
+\langle 1\,4\rangle(-4\langle 2\,3\rangle[2\,4][2\,5][3\,4][3\,5]s_{23}+
$$

$$
+\langle 1\,5\rangle[1\,2][3\,5](3s_{25}-s_{15}-s_{35})[4\,5]^2-2\langle 2\,3\rangle[4\,5]\times
$$

$$
\times([2\,5][2\,3][3\,4](s_{25}-s_{35})+
$$

$$
+\langle 1\,5\rangle([3\,5](-2[1\,4][2\,3][2\,5]+[1\,2](5[2\,5][3\,4]+3[2\,4][3\,5]))+
$$

$$
+[1\,5][2\,3](3[2\,4][3\,5]+[2\,3][4\,5])))))\Big]+
$$

$$
+\langle 1\,2\rangle^2\Big[4[2\,5](\langle 1\,3\rangle^2[1\,2][3\,4]^2[3\,5]\langle 4^-|-2p_2+p_3|5^-\rangle-
$$

$$
-s_{23}\langle 1\,5\rangle[2\,5][4\,5]\langle 3^+|-p_4p_3-p_2p_4|5^-\rangle+
$$

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$$
\begin{aligned}
&+\langle 1\,3\rangle\langle 1\,5\rangle[1\,2][3\,5][4\,5](-s_{34}[3\,5]+\langle 5^+|-p_2p_4+p_4p_2|3^-\rangle))+\\
&+\langle 1\,4\rangle(-4s_{23}^2[2\,5]^2[3\,4][4\,5]+[1\,2][3\,5][4\,5]\times\\
&\times(2\langle 1\,3\rangle^2(-[3\,4][1\,5][2\,3]+[1\,3][2\,4][3\,5])+\\
&+\langle 1\,5\rangle[2\,5](3s_{35}-s_{15}-s_{25})[4\,5]+\\
&+\langle 1\,3\rangle(-s_{35}(3[2\,5][3\,4]+[2\,4][3\,5])+s_{25}([2\,5][3\,4]+3[2\,4][3\,5])\\
&-3s_{15}[2\,3][4\,5]))-\\
&-2\langle 2\,3\rangle([2\,5][4\,5]([2\,3][2\,4][3\,5](s_{35}-s_{25})-\\
&-\langle 1\,5\rangle([1\,5][2\,3](-[2\,5][3\,4]+[2\,3][4\,5])+\\
&+[1\,2][3\,5]([2\,5][3\,4]+3[2\,4][3\,5])))+\\
&+\langle 1\,3\rangle(2[1\,5][2\,3]^2[2\,5][3\,4][4\,5]+\\
&+[1\,2][3\,5](2[2\,5]^2[3\,4]^2-[2\,3][2\,4][3\,5][4\,5]+\\
&+[2\,5][3\,4](4[2\,4][3\,5]+[2\,3][4\,5]))))))\Big]\Big\}+\\
&+\frac{\langle 1\,2\rangle}{\langle 1\,3\rangle[1\,2]s_{15}s_{4H}t_{125}}\times\\
&\times[-\langle 4^+|p_3(p_1+p_2)|5^-\rangle-[4\,5]t_{125}][\langle 1\,3\rangle[3\,4][3\,5]+\langle 1^-|p_2+p_5|3^-\rangle[4\,5]]-\\
&-\frac{1}{\langle 1\,4\rangle\langle 4\,5\rangle[1\,2][2\,5]s_{15}s_{3H}}\times\\
&\times\Big[\langle 1\,2\rangle(\langle 1\,4\rangle[1\,5][2\,4]-[2\,5](s_{12}+s_{15}+s_{24}+s_{25}))[3\,5]\langle 1^-|-p_4-p_5|3^-\rangle-\\
&-[4\,5](\langle 1\,4\rangle^2[3\,4]^2\langle 4^-|-p_1-p_2|5^-\rangle-\\
&-[3\,5](\langle 1\,2\rangle^2\langle 4\,5\rangle[2\,3][2\,5]-\langle 1\,4\rangle\langle 1\,5\rangle\langle 2\,4\rangle[2\,5][3\,4]-[3\,4]\langle 1\,4\rangle^2(-2s_{15}-s_{25})))+\\
&+\langle 1\,5\rangle[3\,5]^2(s_{15}+s_{25})\langle 1\,4\rangle[4\,5]\Big]-\frac{\langle 1\,2\rangle}{\langle 1\,4\rangle[1\,2]s_{15}s_{3H}t_{125}}\times\\
&\times[-\langle 3^+|p_4(p_1+p_2)|5^-\rangle-[3\,5]t_{125}][\langle 1\,2\rangle[2\,4][3\,5]+\langle 1^-|p_4+p_5|3^-\rangle[4\,5]]+\\
&+\frac{\langle 1^-|p_4+p_5|3^-\rangle[4\,5]}{\langle 1\,3\rangle\langle 4\,5\rangle[2\,5]s_{15}s_{3H}t_{145}}[\langle 1\,2\rangle(-s_{25}\langle 1\,3\rangle[3\,5]+\\
&+\langle 1\,3\rangle(-\langle 3^+|p_4(p_1+p_2)|5^-\rangle-[3\,5](s_{12}+s_{15})))-\\
&-\langle 1\,3\rangle\langle 2\,4\rangle[4\,5]\langle 1^-|p_H|3^-\rangle]+\\
&+\frac{\langle 1\,2\rangle^2}{\langle 1\,3\rangle\langle 3\,4\rangle\langle 4\,5\rangle s_{15}}\langle 1^-|p_3+p_4|5^-\rangle-\frac{\langle 1\,2\rangle^2[4\,5]}{\langle 1\,3\rangle\langle 4\,5\rangle s_{15}t_{145}}\langle 1^-|p_4+p_5|3^-\rangle+\\
&+\frac{\langle 1\,2\rangle^3\langle 1^-|p_H|2^-\rangle}{\langle 1\,3\rangle\langle 1\,5\rangle\langle 3\,4\rangle\langle 4\,5\rangle s_{1H}}+\frac{\langle 1\,2\rangle^3}{\langle 1\,3\rangle\langle 1\,4\rangle\langle 1\,5\rangle\langle 2\,3\rangle\langle 4\,5\rangle s_{1H}}\langle 1^-|(p_4+p_5)(p_2+p_3)|1^+\rangle-\\
&-\frac{\langle 1\,2\rangle^3\langle 1^-|p_H|5^-\rangle}{\langle 1\,4\rangle\langle 1\,5\rangle\langle 2\,3\rangle\langle 3\,4\rangle s_{1H}}\Bigg\}. \qquad \text{(B.3)}
\end{aligned}
$$

All the other sub-amplitudes can be obtained by relabelling and by use of reflection symmetry, and parity inversion.

B.2 Sub-amplitudes for Higgs + three gluons and a $q\bar{q}$ pair

The colour decomposition of the amplitudes for Higgs, three gluons and a $q\bar{q}$ pair is given in eq. (2.3), with $n = 5$. There are four independent sub-amplitudes, which we may take to be the one which has no gluons of negative helicity, and the three with one gluon of negative helicity in different positions. They are

$m_5(1_q^+, 2_{\bar{q}}^-; 3^+, 4^+, 5^+) =$

$$= iA\Big\{-\frac{(s_{12}+s_{13})\langle 2^-|(p_1+p_3)p_H|2^+\rangle^2}{s_{12}\langle 1\,3\rangle\langle 2\,3\rangle\langle 2\,4\rangle\langle 2\,5\rangle\langle 4\,5\rangle t_{123}} - \frac{[1\,5]\langle 2^-|(p_1+p_5)p_H|2^+\rangle^2}{s_{12}\langle 2\,3\rangle\langle 2\,4\rangle\langle 2\,5\rangle\langle 3\,4\rangle t_{125}} - \frac{\langle 2^-|p_H|1^-\rangle^2}{\langle 2\,3\rangle\langle 2\,5\rangle\langle 3\,4\rangle\langle 4\,5\rangle[1\,2]} +$$
$$+\frac{\langle 2^-|p_H|5^-\rangle^2}{s_{12}\langle 1\,3\rangle\langle 2\,3\rangle\langle 2\,4\rangle\langle 3\,4\rangle s_{5H}}\Big[-\frac{(s_{12}+s_{13})\langle 2^-|(p_1+p_3)p_4|3^+\rangle}{t_{123}} -$$
$$-\langle 2^-|(p_3+p_4)p_1|3^+\rangle - s_{12}\langle 2\,3\rangle\Big] + \frac{\langle 2^-|p_H|4^-\rangle^2}{s_{12}\langle 1\,3\rangle\langle 2\,3\rangle\langle 2\,5\rangle s_{4H}} \times$$
$$\Big[\frac{[1\,5]\langle 1^-|p_3(p_1+p_5)|2^+\rangle}{t_{125}} - \frac{(s_{12}+s_{13})\langle 2^-|p_1+p_3|5^-\rangle}{t_{123}}\Big] +$$
$$+\frac{\langle 2^-|p_H|3^-\rangle^2}{s_{12}\langle 2\,4\rangle\langle 2\,5\rangle\langle 4\,5\rangle s_{3H}}\Big[\langle 2^-|p_4+p_5|1^-\rangle - \frac{\langle 4\,5\rangle[1\,5]\langle 2^-|p_1+p_5|4^-\rangle}{t_{125}}\Big]\Big\}, \tag{B.4}$$

$$m_5(1_q^+, 2_{\bar q}^-; 3^-, 4^+, 5^+) =$$
$$= iA\Big\{\frac{\langle 2\,3\rangle[4\,5]^2}{s_{34}s_{12}s_{45}}\langle 3^-|p_4+p_5-p_2|1^-\rangle + \frac{\langle 2\,3\rangle[4\,5]}{2\langle 3\,5\rangle s_{34}s_{12}t_{123}} \times$$
$$\times\big[[1\,4]\langle 3^-|p_1(p_4+p_5)+p_2(p_4-p_5)|3^+\rangle + 2\langle 2\,3\rangle(\langle 3\,5\rangle[1\,5][2\,4] - [1\,2]\langle 3^-|p_1+p_2|4^-\rangle)\big] +$$
$$+\frac{\langle 2\,3\rangle[1\,4][4\,5]}{\langle 3\,5\rangle[1\,3]s_{34}t_{123}}\langle 3^-|p_4+p_5-p_2|1^-\rangle - \frac{[4\,5]}{\langle 3\,5\rangle s_{34}s_{12}s_{45}t_{123}} \times$$
$$\times\big[\langle 3^-|p_1+p_2|4^-\rangle(\langle 1\,2\rangle\langle 3^-|p_4+p_5|1^-\rangle^2 - \langle 2\,3\rangle^2[1\,2]s_{45}) +$$
$$+\langle 3^-|p_4+p_5|1^-\rangle\langle 2\,3\rangle\langle 3\,5\rangle[4\,5](s_{13}+s_{23})\big] + \frac{\langle 1\,3\rangle[1\,4][4\,5]}{\langle 3\,5\rangle s_{34}s_{13}s_{45}t_{123}} \times$$
$$\times\big[\langle 1\,2\rangle\langle 3^-|p_4+p_5|1^-\rangle^2 - \langle 2\,3\rangle(s_{34}+s_{35})\langle 3^-|p_4+p_5|1^-\rangle - s_{45}\langle 2\,3\rangle^2[1\,2]\big] -$$
$$-\frac{[4\,5]^2}{s_{34}s_{12}s_{45}t_{345}}\Big[\langle 1\,2\rangle\langle 3^-|p_4+p_5|1^-\rangle^2 - \langle 2\,3\rangle^2[1\,2]t_{345}\Big] + \frac{\langle 2\,3\rangle[1\,4][4\,5]}{2s_{34}s_{12}s_{5H}}\langle 3^-|p_H|5^-\rangle -$$
$$-\frac{1}{2\langle 3\,5\rangle s_{34}s_{12}t_{123}s_{5H}} \times$$
$$\times\Big[2\langle 1\,2\rangle\langle 3\,5\rangle[1\,5]^2\langle 3^-|p_1+p_2|4^-\rangle^2 - 2\langle 2\,3\rangle[1\,4][4\,5]\langle 3^-|(p_1p_3-p_4p_1)p_5p_1|3^+\rangle +$$
$$+\langle 1\,3\rangle\langle 2\,3\rangle\langle 3\,5\rangle[1\,4][1\,5][4\,5](s_{24}-s_{12}-s_{14}) -$$
$$-2\langle 1\,3\rangle\langle 2\,5\rangle[1\,4][1\,5][4\,5]\langle 3^-|(p_1+p_2)p_4|3^+\rangle +$$
$$+\langle 2\,3\rangle\langle 3\,5\rangle[4\,5]^2\langle 3^-|p_4p_2(p_3+p_4)|1^-\rangle + \langle 2\,3\rangle\langle 3\,5\rangle[4\,5]^2\langle 1^+|(p_3p_2+p_4p_3)p_4|3^+\rangle +$$
$$+2\langle 2\,3\rangle[1\,4][4\,5]\langle 3^-|p_2p_5(p_3+p_4)p_1|3^+\rangle + 2\langle 2\,3\rangle^2\langle 3\,5\rangle[1\,5][2\,4][4\,5](s_{13}+s_{23}) +$$
$$+\langle 2\,3\rangle^2\langle 3\,5\rangle[1\,4][2\,5][4\,5](s_{24}+s_{34}-s_{12}-s_{14}) -$$
$$-2\langle 1\,3\rangle\langle 2\,3\rangle^2[1\,2][1\,4][4\,5](s_{35}+s_{45}) - \langle 3\,4\rangle\langle 3\,5\rangle[1\,4][4\,5]^2\langle 3^-|(p_2+p_4)p_1|2^+\rangle +$$
$$+\langle 1\,3\rangle\langle 3\,4\rangle\langle 3\,5\rangle[1\,4][4\,5](\langle 2\,4\rangle[1\,4][4\,5] - \langle 2\,3\rangle[1\,5][3\,4]) +$$
$$+2\langle 1\,3\rangle\langle 3\,5\rangle[1\,4][4\,5](-[1\,5]\langle 3^-|p_1p_4|2^+\rangle + \langle 2\,3\rangle\langle 1^+|p_3p_4|5^-\rangle) +$$
$$+2\langle 2\,3\rangle\langle 3\,5\rangle[2\,4][4\,5](-[1\,5]\langle 2^-|p_1p_4|3^+\rangle + \langle 2\,3\rangle\langle 1^+|p_2p_4|5^-\rangle)\Big] + \frac{\langle 1\,3\rangle[1\,4]}{2s_{34}s_{13}t_{123}s_{5H}} \times$$
$$\times\big[2\langle 2^-|p_H|5^-\rangle(\langle 3\,4\rangle[1\,4][4\,5] + [1\,5]\langle 3^-|p_1+p_2|4^-\rangle) + \langle 3^-|p_H|5^-\rangle\langle 2\,3\rangle[1\,3][4\,5]\big] +$$
$$+\frac{\langle 1\,3\rangle[1\,4]^2}{s_{34}s_{13}t_{134}s_{5H}}\langle 2^-|p_H|5^-\rangle\langle 3^-|p_1+p_4|5^-\rangle - \frac{\langle 2\,3\rangle[1\,4][4\,5]}{2\langle 3\,5\rangle[3\,4]s_{12}s_{4H}}\langle 3^-|p_H|4^-\rangle -$$
$$-\frac{\langle 1\,3\rangle\langle 2\,3\rangle[1\,4]^2[2\,5]}{\langle 3\,5\rangle[3\,4]s_{13}s_{25}s_{4H}}\langle 2^-|p_H|4^-\rangle - \frac{[3\,5]}{2s_{34}s_{12}s_{35}t_{123}s_{4H}} \times$$
$$\times\Big[\langle 3^-|-p_1-p_2|4^-\rangle\langle 3\,4\rangle[1\,4](2\langle 1\,2\rangle\langle 1\,3\rangle[1\,4][1\,5] + \langle 1\,2\rangle\langle 2\,3\rangle[1\,5][2\,4] +$$
$$+\langle 1\,2\rangle\langle 2\,3\rangle[1\,4][2\,5] - \langle 1\,5\rangle\langle 2\,3\rangle[1\,5][4\,5] +$$

$$+ 2\langle 1\,3\rangle\langle 2\,5\rangle[1\,5][4\,5] - s_{35}\langle 2\,3\rangle[4\,5]) -$$

$$- 2\langle 2\,3\rangle[1\,4][4\,5]\langle 3^-|(p_1 p_5 p_2 - p_2 p_3 p_5)p_4|3^+\rangle +$$

$$+ \langle 2\,3\rangle[1\,4][4\,5]\langle 3^-|(p_1 p_4 + p_4 p_2)p_5 p_2|3^+\rangle - \langle 3\,4\rangle\langle 3\,5\rangle[1\,4][4\,5]^2\langle 3^-|(p_2+p_5)p_1|2^+\rangle +$$

$$+ \langle 2\,3\rangle\langle 3\,5\rangle[4\,5]^2\langle 1^+|p_5(p_2+p_3)p_4|3^+\rangle - \langle 2\,3\rangle\langle 3\,5\rangle[4\,5]^2\langle 3^-|p_4 p_5(p_2+p_3)|1^-\rangle +$$

$$+ 2\langle 2\,3\rangle^2[1\,4][4\,5](s_{45}\langle 1\,3\rangle[1\,2] - \langle 3^-|p_5 p_1 p_4|2^-\rangle) +$$

$$+ \langle 1\,3\rangle\langle 3\,5\rangle[1\,4][4\,5](\langle 2\,3\rangle\langle 1^+| - p_4 p_2|5^-\rangle + [1\,5]\langle 2^-|p_5 p_4|3^-\rangle) +$$

$$+ 2\langle 1\,3\rangle\langle 3\,4\rangle\langle 3\,5\rangle[1\,4][4\,5](\langle 1\,2\rangle[1\,4][1\,5] - \langle 2\,3\rangle[1\,3][4\,5])\Big] -$$

$$- \frac{\langle 1\,3\rangle[1\,4]}{2\langle 3\,5\rangle[3\,4]s_{13}t_{123}s_{4H}}\Big[2\langle 3^-| - p_1 - p_2|5^-\rangle[1\,4](\langle 2^-| - p_1 - p_3|4^-\rangle + \langle 2\,5\rangle[4\,5])$$

$$+ \langle 2\,3\rangle[1\,3][4\,5]\langle 3^-|p_H|4^-\rangle + 2\langle 3\,5\rangle[1\,5][4\,5]\langle 2^-|p_H|4^-\rangle\Big] +$$

$$+ \frac{1}{\langle 3\,5\rangle[3\,4]s_{12}t_{125}s_{4H}}\langle 2^-|p_1+p_5|4^-\rangle\langle 3^-|p_H|4^-\rangle\left[\langle 2\,3\rangle[1\,4][2\,5] - [1\,5]\langle 3^-|p_1+p_5|4^-\rangle\right] +$$

$$+ \frac{\langle 2\,3\rangle[1\,4][2\,5]}{\langle 3\,5\rangle[3\,4]s_{25}t_{125}s_{4H}}\langle 2^-|p_1+p_5|4^-\rangle\langle 3^-|p_H|4^-\rangle +$$

$$+ \frac{\langle 2\,3\rangle^2[1\,2][4\,5]}{\langle 3\,4\rangle\langle 3\,5\rangle s_{12}s_{45}s_{3H}}\langle 3^-|(p_1+p_2)(p_4+p_5)|3^+\rangle -$$

$$- \frac{\langle 2\,3\rangle^2[1\,2]}{\langle 3\,4\rangle\langle 3\,5\rangle s_{12}t_{125}s_{3H}}\langle 3^-|p_1+p_2|5^-\rangle\langle 3^-|p_H|4^-\rangle +$$

$$+ \frac{\langle 2\,3\rangle^2[2\,5]}{\langle 3\,4\rangle\langle 3\,5\rangle s_{25}t_{125}s_{3H}}\langle 3^-|p_2+p_5|1^-\rangle\langle 3^-|p_H|4^-\rangle +$$

$$+ \frac{\langle 2\,3\rangle^2[2\,5]}{\langle 3\,4\rangle\langle 3\,5\rangle s_{25}t_{245}s_{3H}}\langle 3^-|p_2+p_5|4^-\rangle\langle 3^-|p_H|1^-\rangle +$$

$$+ \frac{\langle 2\,3\rangle^2[4\,5]}{\langle 3\,4\rangle\langle 3\,5\rangle s_{45}t_{245}s_{3H}}\langle 3^-|p_4+p_5|2^-\rangle\langle 3^-|p_H|1^-\rangle\,, \tag{B.5}$$

$$m_5(1_q^+, 2_{\bar q}^-; 3^+, 4^-, 5^+) =$$

$$= iA\Bigg\{\frac{\langle 2\,4\rangle[3\,5]^2}{s_{45}s_{12}s_{34}}\langle 4^-|p_3+p_5-p_2|1^-\rangle + \frac{\langle 2\,4\rangle[3\,5]}{2\langle 3\,4\rangle s_{45}s_{12}t_{125}}\times$$

$$\times\left[\langle 4^-|p_1+p_2|5^-\rangle\langle 4^-|p_3-2p_2|1^-\rangle - \langle 4^-|p_2-p_5|1^-\rangle\langle 3\,4\rangle[3\,5]\right] +$$

$$+ \frac{\langle 2\,4\rangle^2[2\,5][3\,5]}{\langle 3\,4\rangle s_{45}s_{25}t_{125}}\langle 4^-|p_2-p_3+p_5|1^-\rangle + \frac{[3\,5]}{s_{45}s_{12}s_{34}t_{125}}\times$$

$$\times[-\langle 4^-|p_1+p_2|5^-\rangle(\langle 1\,2\rangle\langle 3\,4\rangle[1\,3]^2 + \langle 2\,4\rangle^2[1\,2][3\,4]) +$$

$$+ \langle 3\,4\rangle[1\,3][1\,5][3\,5](\langle 1\,4\rangle\langle 2\,5\rangle + \langle 1\,2\rangle\langle 4\,5\rangle) +$$

$$+ \langle 2\,5\rangle\langle 3\,4\rangle[3\,5](\langle 2\,4\rangle[1\,3][2\,5] + \langle 4\,5\rangle[1\,5][3\,5])] +$$

$$+ \frac{\langle 2\,4\rangle[2\,5][3\,5]}{s_{45}s_{25}s_{34}t_{125}}\left[[1\,3]\langle 2^-|(-p_1-p_5)p_3|4^+\rangle + \langle 2\,4\rangle\langle 3^+|p_4(p_2+p_5)|1^-\rangle\right]$$

$$- \frac{[3\,5]^2}{s_{45}s_{12}s_{34}t_{345}}\left[\langle 1\,2\rangle\langle 4^-|p_3+p_5|1^-\rangle^2 - \langle 2\,4\rangle^2[1\,2]t_{345}\right] +$$

$$+ \frac{\langle 2\,4\rangle[1\,5][3\,5]}{\langle 3\,4\rangle[4\,5]s_{12}s_{5H}}\langle 4^-|p_H|5^-\rangle + \frac{[3\,5]}{\langle 4\,5\rangle s_{12}s_{34}s_{5H}}\times$$

$$\times[\langle 1\,2\rangle[1\,5]^2\langle 4^-|p_1+p_2|3^-\rangle + [1\,3][1\,5][3\,5](\langle 1\,4\rangle\langle 2\,3\rangle - \langle 1\,2\rangle\langle 3\,4\rangle) -$$

$$- \langle 2\,4\rangle[1\,5][3\,5](s_{14}+s_{23}+s_{24}) - \langle 3\,4\rangle[3\,5]^2\langle 2^-|p_3+p_4|1^-\rangle] +$$

$$
\begin{aligned}
&+\frac{1}{\langle 3\,4\rangle[4\,5]s_{12}t_{123}s_{5H}}\langle 2^-|p_1+p_3|5^-\rangle\langle 4^-|p_H|5^-\rangle\times\\
&\times\left[[1\,3]\langle 4^-|-p_1-p_3|5^-\rangle+\langle 2\,4\rangle[1\,5][2\,3]\right]-\\
&-\frac{[1\,3]}{\langle 3\,4\rangle[4\,5]s_{13}t_{123}s_{5H}}\langle 2^-|p_1+p_3|5^-\rangle\langle 4^-|p_1+p_3|5^-\rangle\langle 4^-|p_H|5^-\rangle-\\
&-\frac{[1\,3]}{\langle 3\,4\rangle[4\,5]s_{13}t_{134}s_{5H}}\langle 4^-|p_1+p_3|5^-\rangle^2\langle 2^-|p_H|5^-\rangle+\\
&+\frac{[1\,3][3\,5]}{[4\,5]s_{34}t_{134}s_{5H}}\langle 4^-|p_1+p_3|5^-\rangle\langle 2^-|p_H|5^-\rangle+\\
&+\frac{\langle 2\,4\rangle^2[1\,3][2\,5]}{\langle 3\,4\rangle\langle 4\,5\rangle s_{13}s_{25}s_{4H}}\langle 4^-|(p_2+p_5)(p_1+p_3)|4^+\rangle-\\
&-\frac{\langle 2\,4\rangle^2[1\,2]}{\langle 3\,4\rangle\langle 4\,5\rangle s_{12}t_{123}s_{4H}}\langle 4^-|p_1+p_2|3^-\rangle\langle 4^-|p_H|5^-\rangle-\\
&-\frac{\langle 2\,4\rangle^2[1\,3]}{\langle 3\,4\rangle\langle 4\,5\rangle s_{13}t_{123}s_{4H}}\langle 4^-|p_1+p_3|2^-\rangle\langle 4^-|p_H|5^-\rangle-\\
&-\frac{\langle 2\,4\rangle^2[1\,2]}{\langle 3\,4\rangle\langle 4\,5\rangle s_{12}t_{125}s_{4H}}\langle 4^-|p_1+p_2|5^-\rangle\langle 4^-|p_H|3^-\rangle+\\
&+\frac{\langle 2\,4\rangle^2[2\,5]}{\langle 3\,4\rangle\langle 4\,5\rangle s_{25}t_{125}s_{4H}}\langle 4^-|p_2+p_5|1^-\rangle\langle 4^-|p_H|3^-\rangle-\\
&-\frac{\langle 2\,4\rangle[1\,5][3\,5]}{2s_{45}s_{12}s_{3H}}\langle 4^-|p_H|3^-\rangle-\frac{1}{2\langle 3\,4\rangle s_{45}s_{12}t_{125}s_{3H}}\times\\
&\times\Big[2\langle 1\,2\rangle\langle 3\,4\rangle[1\,3]^2\langle 4^-|p_1+p_2|5^-\rangle^2-\langle 2\,4\rangle\langle 3\,4\rangle[1\,5][3\,5]\langle 4^-|p_1+p_5|3^-\rangle(s_{12}-s_{14})-\\
&\quad-2(\langle 2\,5\rangle\langle 3\,4\rangle-\langle 2\,3\rangle\langle 4\,5\rangle)\langle 1\,4\rangle[1\,3][1\,5][3\,5]\langle 4^-|p_1+p_2|5^-\rangle-2\langle 4^-|p_1+p_2|5^-\rangle\times\\
&\quad\times(-s_{13}\langle 2\,4\rangle+\langle 2\,5\rangle\langle 3\,4\rangle[3\,5])\langle 4\,5\rangle[1\,5][3\,5]-\\
&\qquad-\langle 2\,4\rangle[1\,5][3\,5]\langle 4^-|p_2p_1(p_2+p_4)p_3|4^+\rangle+2\langle 2\,4\rangle\langle 3\,4\rangle[1\,3][3\,5]\langle 5^+|p_2(p_4+p_5)p_1|4^+\rangle-\\
&\qquad-\langle 2\,4\rangle^2\langle 3\,4\rangle[1\,3][2\,5][3\,5](s_{24}+2s_{25})-\langle 2\,4\rangle^2\langle 3\,4\rangle[3\,5]^2\langle 1^+|p_4p_5+p_2p_4|2^-\rangle+\\
&\qquad+\langle 2\,4\rangle\langle 3\,4\rangle\langle 4\,5\rangle[3\,5][4\,5](\langle 1\,4\rangle[1\,3][1\,5]+\langle 2\,4\rangle[1\,3][2\,5]+\langle 4\,5\rangle[1\,5][3\,5])\Big]-\\
&-\frac{\langle 2\,4\rangle[2\,5]}{2s_{45}s_{25}t_{125}s_{3H}}\Big[-2\langle 1\,2\rangle[1\,3]^2\langle 4^-|p_1+p_2|5^-\rangle+\langle 2\,5\rangle[1\,5][3\,5]\langle 4^-|p_H|3^-\rangle-\\
&\qquad\qquad-2[1\,3][3\,5](\langle 2^-|(p_4+p_5)p_1|4^+\rangle+\langle 2\,4\rangle t_{245})\Big]-\\
&-\frac{\langle 2\,4\rangle^2[1\,3][2\,5]}{s_{45}s_{25}t_{245}s_{3H}}[-\langle 3^+|p_1(p_2+p_4)|5^-\rangle-[3\,5]t_{245}]\Big\}\,,
\end{aligned}
\tag{B.6}
$$

$$
\begin{aligned}
&m_5(1_q^+,2_{\bar q}^-;3^+,4^+,5^-)=\\
&=iA\Big\{\frac{\langle 2\,5\rangle[3\,4]^2}{s_{12}s_{34}s_{45}}\langle 5^-|p_3+p_4-p_2|1^-\rangle+\frac{\langle 2\,5\rangle[3\,4]}{2\langle 3\,5\rangle s_{12}s_{45}t_{125}}\times\\
&\quad\times\left[\langle 5^-|2p_2-p_3-p_4|1^-\rangle\langle 5^-|-p_1-p_2|4^-\rangle-\langle 2\,5\rangle\langle 3\,5\rangle[1\,2][3\,4]\right]+\\
&\quad+\frac{\langle 2\,5\rangle^2[2\,4][3\,4]}{\langle 3\,5\rangle s_{25}s_{45}t_{125}}\langle 5^-|p_2-p_3-p_4|1^-\rangle+\frac{[3\,4]}{\langle 3\,5\rangle s_{12}s_{34}s_{45}t_{125}}\times\\
&\quad\times\Big[\langle 1\,2\rangle\langle 1\,5\rangle[1\,4]\langle 5^-|p_3+p_4|1^-\rangle^2+\langle 2\,5\rangle\langle 3\,5\rangle[1\,3][2\,4]\times\\
&\qquad\times(\langle 2^-|-p_1p_3|5^+\rangle-\langle 2\,5\rangle s_{14})+\langle 2\,5\rangle^2[1\,4](\langle 5^-|p_1p_4p_3|2^-\rangle-s_{13}\langle 5^-|p_4|2^-\rangle)-\\
&\qquad-\langle 2\,5\rangle\langle 3\,5\rangle[3\,4](s_{15}+s_{25})\langle 5^-|p_3+p_4|1^-\rangle+\langle 2\,5\rangle\langle 4\,5\rangle[1\,4][2\,4]\langle 5^-|-2p_1p_3+p_4p_1|2^+\rangle-
\end{aligned}
$$

$$
\begin{aligned}
&\quad - \langle 2\,5\rangle^3[1\,2][2\,4]s_{34}\Big] + \frac{\langle 2\,5\rangle[3\,4]}{\langle 3\,5\rangle s_{25}s_{34}s_{45}t_{125}} \times \\
&\times \Big[\langle 5^-|p_3+p_4|1^-\rangle^2\langle 1^-|p_2+p_5|4^-\rangle - \langle 2\,5\rangle^2[1\,2][2\,4]s_{34} - s_{25}\langle 3\,5\rangle[3\,4]\langle 5^-|p_3+p_4|1^-\rangle\Big] - \\
&- \frac{[3\,4]^2}{s_{12}s_{34}s_{45}t_{345}}\Big[\langle 1\,2\rangle\langle 5^-|p_3+p_4|1^-\rangle^2 - t_{345}\langle 2\,5\rangle^2[1\,2]\Big] - \\
&- \frac{\langle 2\,5\rangle^2[1\,2][3\,4]}{\langle 3\,5\rangle\langle 4\,5\rangle s_{12}s_{34}s_{5H}}\langle 5^-|(p_1+p_2)(p_3+p_4)|5^+\rangle - \\
&- \frac{\langle 2\,5\rangle^2[1\,2]}{\langle 3\,5\rangle\langle 4\,5\rangle s_{12}s_{5H}t_{123}}\langle 5^-|p_1+p_2|3^-\rangle\langle 5^-|p_H|4^-\rangle - \\
&- \frac{\langle 2\,5\rangle^2[1\,3]}{\langle 3\,5\rangle\langle 4\,5\rangle s_{13}s_{5H}t_{123}}\langle 5^-|p_1+p_3|2^-\rangle\langle 5^-|p_H|4^-\rangle - \\
&- \frac{\langle 2\,5\rangle^2[1\,3]}{\langle 3\,5\rangle\langle 4\,5\rangle s_{13}s_{5H}t_{134}}\langle 5^-|p_1+p_3|4^-\rangle\langle 5^-|p_H|2^-\rangle + \\
&+ \frac{\langle 2\,5\rangle^2[3\,4]}{\langle 3\,5\rangle\langle 4\,5\rangle s_{34}s_{5H}t_{134}}\langle 5^-|p_3+p_4|1^-\rangle\langle 5^-|p_H|2^-\rangle + \frac{\langle 2\,5\rangle[1\,4][3\,4]}{2\langle 3\,5\rangle[4\,5]s_{12}s_{4H}}\langle 5^-|p_H|4^-\rangle - \\
&- \frac{\langle 2\,5\rangle[1\,3]}{\langle 3\,5\rangle[4\,5]s_{13}s_{25}s_{4H}}\langle 5^-|p_1+p_3|4^-\rangle\langle 4^+|(p_1+p_3)(p_2+p_5)|4^-\rangle + \\
&+ \frac{1}{\langle 3\,5\rangle[4\,5]s_{12}s_{4H}t_{123}}\langle 2^-|p_1+p_3|4^-\rangle\langle 5^-|p_H|4^-\rangle[-[1\,4]\langle 5^-|p_1+p_2|3^-\rangle + \langle 3\,5\rangle[1\,3][3\,4]] - \\
&- \frac{[1\,3]}{\langle 3\,5\rangle[4\,5]s_{13}t_{123}s_{4H}}\langle 2^-|p_1+p_3|4^-\rangle\langle 5^-|p_1+p_3|4^-\rangle\langle 5^-|p_H|4^-\rangle - \\
&- \frac{1}{2\langle 3\,5\rangle s_{45}s_{12}t_{125}s_{4H}}\Big[\langle 4\,5\rangle[1\,4]\langle 5^-|p_1+p_2|4^-\rangle(2\langle 1\,2\rangle\langle 1\,5\rangle[1\,3][1\,4] + \langle 1\,2\rangle\langle 2\,5\rangle[1\,4][2\,3] + \\
&\qquad + \langle 1\,2\rangle\langle 2\,5\rangle[1\,3][2\,4] - 2\langle 1\,5\rangle\langle 2\,3\rangle[1\,3][3\,4] - s_{13}\langle 2\,5\rangle[3\,4]) + \\
&\qquad + \langle 2\,5\rangle[1\,4][3\,4]\langle 5^-|p_1(p_4p_2p_3+p_3p_4p_2)|5^+\rangle - \\
&\qquad - 2\langle 1\,5\rangle\langle 2\,3\rangle[1\,3][1\,4][3\,4]\langle 5^-|(p_2+p_3)p_4|5^+\rangle + \\
&\qquad + \langle 2\,5\rangle^2[1\,4][3\,4](-s_{23}\langle 4\,5\rangle[2\,4] - s_{34}\langle 1\,5\rangle[1\,2]) + \\
&\qquad + \langle 2\,5\rangle\langle 3\,5\rangle\langle 4\,5\rangle[1\,4][3\,4]^2(s_{12}+s_{13}+s_{15}) + \\
&\qquad + \langle 2\,5\rangle\langle 3\,5\rangle\langle 4\,5\rangle[3\,4]^2(s_{25}[1\,4] - \langle 1^+|p_3p_2|4^-\rangle) + \\
&\qquad + \langle 2\,5\rangle\langle 3\,5\rangle\langle 4\,5\rangle[3\,4]^3\langle 3^-|p_2+p_5|1^-\rangle + \\
&\qquad + s_{45}\langle 2\,5\rangle[3\,4][1\,4]\langle 5^-|(p_1+p_2)p_3|5^+\rangle - \\
&\qquad - 2\langle 3\,5\rangle[1\,3][1\,4][3\,4](\langle 1\,2\rangle\langle 1\,5\rangle\langle 4\,5\rangle[1\,4] + \langle 1\,4\rangle\langle 2\,5\rangle^2[2\,4]) - \\
&\qquad - s_{45}\langle 2\,5\rangle\langle 3\,5\rangle^2[1\,3][3\,4]^2\Big] + \\
&+ \frac{\langle 2\,5\rangle}{2\langle 3\,5\rangle[4\,5]s_{25}s_{4H}t_{125}}\Big[2\langle 1\,2\rangle[1\,4]^2[2\,4]\langle 5^-|p_1+p_2|3^-\rangle + 2(\langle 1\,5\rangle\langle 2\,3\rangle - \langle 1\,2\rangle\langle 3\,5\rangle) \times \\
&\qquad \times [1\,3][1\,4][2\,4][3\,4] - \langle 2\,5\rangle[1\,4][2\,4][3\,4](s_{15}+s_{25}) + \\
&\qquad + 2\langle 3\,5\rangle[1\,3][3\,4]\langle 4^+|p_1p_5 - p_2p_3|4^-\rangle - \\
&\qquad - 2\langle 3\,5\rangle[1\,3][3\,4]\langle 4^+|p_5(p_1+p_3)|4^-\rangle + \\
&\qquad + \langle 2\,5\rangle[1\,4][2\,3]\langle 4^+|p_1p_5 - p_2p_3|4^-\rangle + \\
&\qquad + \langle 2\,5\rangle[1\,4][2\,4]\langle 3^+|p_1p_5 - p_2p_3|4^-\rangle + \\
&\qquad + \langle 2\,5\rangle\langle 3\,5\rangle[3\,4]([1\,5][2\,4][3\,4] + [1\,4][2\,3][4\,5] + [1\,3][2\,4][4\,5]) + \\
&\qquad + 2\langle 1\,5\rangle^2[1\,3][1\,4]^2[4\,5]\Big] -
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{\langle 2\,5\rangle[1\,4][3\,4]}{2s_{12}s_{45}s_{3H}}\langle 5^-|p_H|3^-\rangle+\frac{1}{2\langle 3\,5\rangle s_{12}s_{45}s_{3H}t_{125}}\times\\
&\times\Big[2\langle 1\,2\rangle\langle 1\,5\rangle\langle 3\,5\rangle[1\,3]^2[1\,4]\langle 5^-|p_1+p_2|4^-\rangle-\langle 1\,2\rangle\langle 1\,5\rangle\langle 2\,5\rangle\langle 3\,5\rangle[1\,3][1\,4]\times\\
&\quad\times([1\,4][2\,3]+[1\,3][2\,4])+\langle 1\,5\rangle\langle 3\,5\rangle[1\,3][1\,4][3\,4]\langle 2^-|2p_5p_1-3p_1p_4|5^+\rangle+\\
&\quad+\langle 1\,5\rangle\langle 2\,4\rangle[1\,4]^2[3\,4]\langle 5^-|(p_1+2p_2)p_3|5^+\rangle-\langle 2\,5\rangle^2\langle 3\,5\rangle[1\,4][2\,3][3\,4](s_{12}+s_{14})-\\
&\quad-2\langle 2\,5\rangle[1\,4][3\,4]\langle 5^-|p_1(p_5p_2p_3+p_3p_2p_4)|5^+\rangle+\\
&\quad+2\langle 2\,5\rangle^2[1\,3][2\,4]\langle 4^+|p_3(p_1p_4-p_5p_1)|5^+\rangle+\\
&\quad+\langle 2\,4\rangle\langle 2\,5\rangle[1\,4][2\,4][3\,4]\langle 5^-|(p_1-p_2)p_3|5^+\rangle-2\langle 2\,5\rangle^3[2\,4][3\,4]\langle 1^+|p_5p_3-p_3p_4|2^-\rangle-\\
&\quad-2\langle 2\,5\rangle^2[1\,3][2\,4]^2\langle 2^-|-p_1p_3+p_3p_4|5^+\rangle+\\
&\quad+\langle 3\,5\rangle\langle 4\,5\rangle[1\,4][3\,4]^2\langle 2^-|p_1(p_2+p_4)-(p_4+p_5)p_1|5^+\rangle+\\
&\quad+\langle 2\,5\rangle[1\,4][3\,4]\langle 5^-|p_4(p_1p_5-p_5p_2)p_3|5^+\rangle-\langle 2\,5\rangle\langle 3\,5\rangle\langle 4\,5\rangle[3\,4]^2[1\,4](s_{24}+s_{45})+\\
&\quad+2\langle 1\,5\rangle\langle 2\,5\rangle[1\,4][3\,4]\langle 1^+|p_2(p_4+p_5)p_3|5^+\rangle+2\langle 2\,5\rangle^2[2\,4][3\,4]\langle 1^+|(p_5p_4+p_2p_5)p_3|5^+\rangle\Big]+\\
&+\frac{\langle 2\,5\rangle}{2s_{25}s_{45}s_{3H}t_{125}}\Big[2\langle 1\,2\rangle[1\,3]^2[2\,4]\langle 5^-|p_1+p_2|4^-\rangle+\\
&\qquad+2[1\,3][1\,4][2\,4][3\,4](\langle 1\,2\rangle\langle 4\,5\rangle-\langle 1\,5\rangle\langle 2\,4\rangle)+\\
&\qquad+\langle 2\,5\rangle[2\,4][3\,4]\langle 1^+|p_5(p_1+p_2)|3^-\rangle-2[1\,3][3\,4]\times\\
&\qquad\times\langle 5^-|p_2(p_4p_2+p_5p_1)|4^-\rangle+2\langle 4\,5\rangle[1\,4][3\,4]\langle 4^+|p_2(p_4+p_5)|3^-\rangle+\\
&\qquad+\langle 2\,5\rangle[1\,3][2\,4]\langle 3^+|2p_5p_2+p_1p_5+p_2p_5|4^-\rangle+\\
&\qquad+\langle 1\,5\rangle[1\,3][1\,4]\langle 4^+|p_5(-2p_1-p_2)|3^-\rangle+\\
&\qquad+\langle 2\,5\rangle[2\,3][1\,4]\langle 4^+|p_5(-2p_4-p_2)|3^-\rangle-2s_{45}[1\,3][3\,4]\langle 5^-|2p_1+p_2|4^-\rangle-\\
&\qquad-\langle 4\,5\rangle[1\,4][3\,4]^2(2s_{45}+s_{25})\Big]+\\
&+\frac{\langle 2\,5\rangle^2[1\,3][2\,4]}{s_{25}s_{45}s_{3H}t_{245}}\left[\langle 3^+|p_1(p_2+p_5)|4^-\rangle+t_{245}[3\,4]\right]\Bigg\}\,, \qquad\qquad (B.7)
\end{aligned}
$$

All the other sub-amplitudes can be obtained by relabelling and by use of reflection symmetry, parity inversion and charge conjugation.

B.3 Sub-amplitudes for Higgs + one gluon and two $q\bar{q}$ pairs

The colour decomposition of the amplitudes for Higgs, one gluon and two $q\bar{q}$ pairs is given in eq. (2.4), with $n=5$. It can be explicitly re-written as,

$$
\begin{aligned}
&\mathcal{M}_5(1_q,2_{\bar{q}};3_Q,4_{\overline{Q}};5)=\\
&=g^3\frac{1}{\sqrt{2}}\bigg[(T^{a_5})_{i_1}^{\,\bar{\imath}_4}\,\delta_{i_3}^{\bar{\imath}_2}m_{5,1}(1_q,2_{\bar{q}};3_Q,4_{\overline{Q}};5)+(T^{a_5})_{i_3}^{\,\bar{\imath}_2}\,\delta_{i_1}^{\bar{\imath}_4}m_{5,2}(1_q,2_{\bar{q}};3_Q,4_{\overline{Q}};5)+ \qquad (B.8)\\
&\qquad+\frac{1}{N_c}(T^{a_5})_{i_1}^{\,\bar{\imath}_2}\,\delta_{i_3}^{\bar{\imath}_4}m_{5,3}(1_q,2_{\bar{q}};3_Q,4_{\overline{Q}};5)+\frac{1}{N_c}(T^{a_5})_{i_3}^{\,\bar{\imath}_4}\,\delta_{i_1}^{\bar{\imath}_2}m_{5,4}(1_q,2_{\bar{q}};3_Q,4_{\overline{Q}};5)\bigg]\,.
\end{aligned}
$$

For each sub-amplitude of type $m_{5,i}$, with $i=1,2,3,4$, there are two independent sub-amplitudes. The two independent sets of four sub-amplitudes can be taken to be

$$
\begin{aligned}
&m_{5,1}(1_q^+,2_{\bar{q}}^-;3_Q^+,4_{\bar{Q}}^-;5^+)=\\
&=iA\bigg\{\frac{1}{\langle 1\,5\rangle s_{12}s_{34}}\left[\langle 2\,4\rangle[3\,5](s_{12}-s_{15})-\langle 2\,4\rangle\langle 1\,4\rangle[1\,5][3\,4]-\langle 1\,2\rangle[1\,3]\langle 4^-|p_1+p_3|5^-\rangle\right]-
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{1}{\langle 1\,5\rangle s_{12} s_{34} t_{125}} \Big[- \langle 1\,2\rangle^2 \langle 3\,4\rangle [1\,3]^2 [2\,5] - \langle 1\,4\rangle\langle 2\,4\rangle[1\,5][3\,4](s_{15}+s_{25}) + \\
&\qquad + s_{12}\langle 2\,4\rangle^2[2\,5][3\,4] + \langle 1\,2\rangle\langle 3\,4\rangle[1\,3][3\,5](s_{25}-s_{15}) - s_{15}\langle 2\,5\rangle\langle 3\,4\rangle[3\,5]^2\Big] + \\
&+ \frac{1}{\langle 1\,5\rangle s_{34} t_{125}} \left[\langle 2\,4\rangle[4\,3]\langle 4^-|p_1+p_2|5^-\rangle + \langle 4\,3\rangle[3\,5]\langle 2^-|p_1+p_5|3^-\rangle\right] - \\
&- \frac{1}{\langle 1\,5\rangle s_{12} s_{34} t_{345}} \Big[- \langle 1\,2\rangle[3\,5](s_{13}+s_{15})\langle 4^-|p_3+p_5|1^-\rangle + \langle 2\,4\rangle^2[1\,2][3\,4]\langle 1^-|p_3+p_4|5^-\rangle \\
&\qquad + \langle 2\,4\rangle[3\,5]s_{12}(s_{35}+s_{45}) + \langle 1\,2\rangle\langle 1\,4\rangle[1\,3]\langle 1^+|(p_3+p_5)p_4|5^-\rangle\Big] - \\
&- \frac{\langle 1\,4\rangle}{\langle 1\,5\rangle\langle 4\,5\rangle s_{12} t_{345}} \left[-\langle 1\,2\rangle[1\,3]\langle 4^-|p_3+p_5|1^-\rangle + \langle 2\,4\rangle[1\,2]\langle 2^-|p_4+p_5|3^-\rangle\right] - \\
&- \frac{1}{s_{12}s_{34}s_{5H}} \Big[- \langle 1\,2\rangle[1\,5]^2\langle 4^-|p_1+p_2|3^-\rangle + [1\,5][3\,5]\langle 2^-|-p_1p_3+p_3p_1|4^+\rangle + \\
&\qquad + \langle 2\,4\rangle[1\,5][3\,5](s_{14}+s_{23}+s_{24}) + \langle 3\,4\rangle[3\,5]^2\langle 2^-|p_3+p_4|1^-\rangle\Big] + \\
&+ \frac{[1\,3]}{s_{12}t_{123}s_{5H}}\langle 2^-|p_1+p_3|5^-\rangle\langle 4^-|p_H|5^-\rangle + \\
&+ \frac{\langle 2\,4\rangle[1\,5]}{s_{34}s_{5H}}\left[\frac{[5\,1]\langle 1^-|p_2+p_4|3^-\rangle}{t_{234}} + [5\,3]\right]\Bigg\}
\end{aligned} \tag{B.9}
$$

$$
\begin{aligned}
&m_{5,2}(1_q^+,2_{\bar q}^-;3_Q^+,4_{\bar Q}^-;5^+) = \\
&= iA\Bigg\{ - \frac{1}{\langle 1\,5\rangle s_{12}s_{34}} \left[\langle 2\,4\rangle[3\,5](s_{12}-s_{15}) - \langle 2\,4\rangle\langle 1\,4\rangle[1\,5][3\,4] - \langle 1\,2\rangle[1\,3]\langle 4^-|p_1+p_3|5^-\rangle\right] + \\
&\quad + \frac{1}{\langle 1\,5\rangle s_{12} s_{34} t_{125}} \Big[- \langle 1\,2\rangle^2 \langle 3\,4\rangle [1\,3]^2 [2\,5] - \langle 1\,4\rangle\langle 2\,4\rangle[1\,5][3\,4](s_{15}+s_{25}) + \\
&\qquad + s_{12}\langle 2\,4\rangle^2[2\,5][3\,4] + \langle 1\,2\rangle\langle 3\,4\rangle[1\,3][3\,5](s_{25}-s_{15}) - s_{15}\langle 2\,5\rangle\langle 3\,4\rangle[3\,5]^2\Big] - \\
&\quad - \frac{\langle 1\,2\rangle}{\langle 1\,5\rangle\langle 2\,5\rangle s_{34}t_{125}} \left[[1\,3]\langle 2^-|(p_1+p_5)p_3|4^+\rangle - \langle 2\,4\rangle\langle 3^+|p_4(p_2+p_5)|1^-\rangle\right] + \\
&\quad + \frac{1}{\langle 1\,5\rangle s_{12} s_{34} t_{345}} \Big[- \langle 1\,2\rangle[3\,5](s_{13}+s_{15})\langle 4^-|p_3+p_5|1^-\rangle + \langle 2\,4\rangle^2[1\,2][3\,4]\langle 1^-|p_3+p_4|5^-\rangle + \\
&\qquad + \langle 2\,4\rangle[3\,5]s_{12}(s_{35}+s_{45}) + \langle 1\,2\rangle\langle 1\,4\rangle[1\,3]\langle 1^+|(p_3+p_5)p_4|5^-\rangle\Big] - \\
&\quad - \frac{1}{\langle 1\,5\rangle\langle 3\,5\rangle s_{12}t_{345}}\Big[(s_{13}+s_{15})\langle 2^-|p_1(p_3+p_5)|4^+\rangle + \langle 2\,4\rangle^2[1\,2]\langle 1^-|(p_3+p_5)|4^-\rangle + \\
&\qquad + \langle 2\,4\rangle s_{12}s_{35}\Big] + \\
&\quad + \frac{1}{s_{12}s_{34}s_{5H}} \Big[- \langle 1\,2\rangle[1\,5]^2\langle 4^-|p_1+p_2|3^-\rangle + [1\,5][3\,5]\langle 2^-|-p_1p_3+p_3p_1|4^+\rangle + \\
&\qquad + \langle 2\,4\rangle[1\,5][3\,5](s_{14}+s_{23}+s_{24}) + \langle 3\,4\rangle[3\,5]^2\langle 2^-|p_3+p_4|1^-\rangle\Big] + \\
&\quad + \frac{\langle 2\,4\rangle[3\,5]}{s_{12}t_{124}s_{5H}}\left[[1\,5]t_{124} + [3\,5]\langle 3^-|p_2+p_4|1^-\rangle\right] - \\
&\quad - \frac{[1\,3]}{s_{34}t_{134}s_{5H}}\langle 4^-|p_1+p_3|5^-\rangle\langle 2^-|p_H|5^-\rangle\Bigg\},
\end{aligned} \tag{B.10}
$$

$$
\begin{aligned}
&m_{5,3}(1_q^+,2_{\bar q}^-;3_Q^+,4_{\bar Q}^-;5^+) = \\
&= iA\Bigg\{ - \frac{1}{\langle 1\,5\rangle s_{34} t_{125}} \left[\langle 2\,4\rangle[4\,3]\langle 4^-|p_1+p_2|5^-\rangle + \langle 4\,3\rangle[3\,5]\langle 2^-|p_1+p_5|3^-\rangle\right] +
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{\langle 1\,2\rangle}{\langle 1\,5\rangle\langle 2\,5\rangle s_{34}t_{125}}\left[[1\,3]\langle 2^-|(p_1+p_5)p_3|4^+\rangle-\langle 2\,4\rangle\langle 3^+|p_4(p_2+p_5)|1^-\rangle\right]+\\
&+\frac{[1\,3]}{s_{34}t_{134}s_{5H}}\langle 4^-|p_1+p_3|5^-\rangle\langle 2^-|p_H|5^-\rangle-\\
&-\frac{\langle 2\,4\rangle[1\,5]}{s_{34}s_{5H}}\left[\frac{[5\,1]\langle 1^-|p_2+p_4|3^-\rangle}{t_{234}}+[5\,3]\right]\Bigg\}\,,
\end{aligned}
\tag{B.11}
$$

$$
\begin{aligned}
&m_{5,4}(1_q^+,2_{\bar q}^-;3_Q^+,4_{\bar Q}^-;5^+)=\\
&=iA\Bigg\{\frac{1}{\langle 1\,5\rangle\langle 3\,5\rangle s_{12}t_{345}}\Big[(s_{13}+s_{15})\langle 2^-|p_1(p_3+p_5)|4^+\rangle+\langle 2\,4\rangle^2[1\,2]\langle 1^-|(p_3+p_5)|4^-\rangle+\\
&\qquad\qquad+\langle 2\,4\rangle s_{12}s_{35}\Big]+\\
&\quad+\frac{\langle 1\,4\rangle}{\langle 1\,5\rangle\langle 4\,5\rangle s_{12}t_{345}}\left[-\langle 1\,2\rangle[1\,3]\langle 4^-|p_3+p_5|1^-\rangle+\langle 2\,4\rangle[1\,2]\langle 2^-|p_4+p_5|3^-\rangle\right]+\\
&\quad-\frac{[1\,3]}{s_{12}t_{123}s_{5H}}\langle 2^-|p_1+p_3|5^-\rangle\langle 4^-|p_H|5^-\rangle\\
&\quad-\frac{\langle 2\,4\rangle[3\,5]}{s_{12}t_{124}s_{5H}}\left[[1\,5]t_{124}+[3\,5]\langle 3^-|p_2+p_4|1^-\rangle\right]\Bigg\}\,.
\end{aligned}
\tag{B.12}
$$

$$
\begin{aligned}
&m_{5,1}(1_q^+,2_{\bar q}^-;3_Q^-,4_{\bar Q}^+;5^+)=\\
&=iA\Bigg\{\frac{1}{\langle 1\,5\rangle s_{12}s_{34}t_{345}}\Big[\langle 2\,3\rangle[1\,2](-\langle 2\,3\rangle[4\,5]\langle 1^-|p_4+p_5|3^-\rangle+\langle 1\,3\rangle[3\,5]\langle 2^-|p_3+p_5|4^-\rangle)+\\
&\qquad\qquad+\langle 1\,2\rangle\langle 3^-|p_4+p_5|1^-\rangle(-\langle 1\,3\rangle[1\,4][3\,5]+[4\,5](s_{14}+s_{15}))-\\
&\qquad\qquad-s_{12}s_{45}\langle 2\,3\rangle[4\,5]\Big]-\\
&\quad-\frac{1}{s_{12}s_{34}s_{5H}}\Big[-\langle 1\,2\rangle[1\,5]^2\langle 3^-|p_1+p_2|4^-\rangle-\\
&\qquad\qquad-[1\,5][4\,5](2\langle 2^-|p_1p_4|3^+\rangle-\langle 2\,3\rangle(s_{13}+s_{14}+s_{23}+s_{24}))-\\
&\qquad\qquad-\langle 3\,4\rangle[4\,5]^2\langle 2^-|p_3+p_4|1^-\rangle\Big]+\\
&\quad+\frac{1}{\langle 1\,5\rangle s_{12}s_{34}t_{125}}\Big[-\langle 1\,2\rangle^2\langle 3\,4\rangle[1\,4]^2[2\,5]+\\
&\qquad\qquad+\langle 1\,2\rangle(-\langle 2\,3\rangle^2[1\,2][2\,5][3\,4]+\langle 3\,4\rangle[1\,4][4\,5](s_{25}-s_{15}))+\\
&\qquad\qquad+[1\,5](-\langle 1\,3\rangle\langle 2\,3\rangle[3\,4](s_{15}+s_{25})+\langle 1\,5\rangle\langle 2\,5\rangle\langle 3\,4\rangle[4\,5]^2)\Big]+\\
&\quad+\frac{1}{\langle 1\,5\rangle s_{12}s_{34}}\Big[\langle 2\,3\rangle[1\,5]\langle 1^-|p_3-p_5|4^-\rangle+\langle 1\,2\rangle(\langle 1\,3\rangle[1\,4][1\,5]-[4\,5]\langle 3^-|p_2-p_4|1^-\rangle)\Big]-\\
&\quad-\frac{1}{\langle 1\,5\rangle\langle 4\,5\rangle s_{12}t_{345}}\Big[-\langle 2\,3\rangle^2[1\,2]\langle 1^-|p_4+p_5|3^-\rangle+\\
&\qquad\qquad+\langle 1\,2\rangle(s_{14}+s_{15})\langle 3^-|p_4+p_5|1^-\rangle-s_{12}s_{45}\langle 2\,3\rangle\Big]+\\
&\quad+\frac{\langle 2\,3\rangle[4\,5]}{s_{12}t_{123}s_{5H}}\left[-[1\,5]t_{123}-[4\,5]\langle 4^-|p_2+p_3|1^-\rangle\right]+\\
&\quad+\frac{\langle 2\,3\rangle[1\,5]}{s_{34}t_{234}s_{5H}}\left[-[1\,5]\langle 1^-|p_2+p_3|4^-\rangle-[4\,5]t_{234}\right]+\\
&\quad+\frac{1}{\langle 1\,5\rangle s_{34}t_{125}}\left[\langle 2\,3\rangle[3\,4]\langle 3^-|p_1+p_2|5^-\rangle+\langle 3\,4\rangle[4\,5]\langle 2^-|p_1+p_5|4^-\rangle\right]\Bigg\}\,,
\end{aligned}
\tag{B.13}
$$

$$
m_{5,2}(1_q^+,2_{\bar q}^-;3_Q^-,4_{\bar Q}^+;5^+)=
$$

$$
\begin{aligned}
= iA\Big\{ &- \frac{1}{\langle 1\,5\rangle s_{12}s_{34}t_{345}}\Big[\langle 2\,3\rangle[1\,2](-\langle 2\,3\rangle[4\,5]\langle 1^-|p_4+p_5|3^-\rangle + \langle 1\,3\rangle[3\,5]\langle 2^-|p_3+p_5|4^-\rangle) + \\
&\qquad + \langle 1\,2\rangle\langle 3^-|p_4+p_5|1^-\rangle(-\langle 1\,3\rangle[1\,4][3\,5] + [4\,5](s_{14}+s_{15})) - \\
&\qquad - s_{12}s_{45}\langle 2\,3\rangle[4\,5]\Big] + \frac{1}{s_{12}s_{34}s_{5H}} \times \\
&\times \Big[- \langle 1\,2\rangle[1\,5]^2\langle 3^-|p_1+p_2|4^-\rangle \\
&\quad - [1\,5][4\,5](2\langle 2^-|p_1p_4|3^+\rangle - \langle 2\,3\rangle(s_{13}+s_{14}+s_{23}+s_{24})) - \langle 3\,4\rangle[4\,5]^2\langle 2^-|p_3+p_4|1^-\rangle\Big] - \\
&- \frac{1}{\langle 1\,5\rangle s_{12}s_{34}t_{125}}\Big[- \langle 1\,2\rangle^2\langle 3\,4\rangle[1\,4]^2[2\,5] + \\
&\qquad + \langle 1\,2\rangle(-\langle 2\,3\rangle^2[1\,2][2\,5][3\,4] + \langle 3\,4\rangle[1\,4][4\,5](s_{25}-s_{15})) + \\
&\qquad + [1\,5](-\langle 1\,3\rangle\langle 2\,3\rangle[3\,4](s_{15}+s_{25}) + \langle 1\,5\rangle\langle 2\,5\rangle\langle 3\,4\rangle[4\,5]^2)\Big] - \\
&- \frac{1}{\langle 1\,5\rangle s_{12}s_{34}}\left[\langle 2\,3\rangle[1\,5]\langle 1^-|p_3-p_5|4^-\rangle + \langle 1\,2\rangle(\langle 1\,3\rangle[1\,4][1\,5] - [4\,5]\langle 3^-|p_2-p_4|1^-\rangle)\right] - \\
&- \frac{[1\,4]}{s_{12}t_{124}s_{5H}}\langle 2^-|p_1+p_4|5^-\rangle\langle 3^-|p_H|5^-\rangle + \\
&+ \frac{\langle 1\,3\rangle}{\langle 1\,5\rangle\langle 3\,5\rangle s_{12}t_{345}}\left[-\langle 1\,2\rangle[1\,4]\langle 3^-|p_4+p_5|1^-\rangle + \langle 2\,3\rangle[1\,2]\langle 2^-|p_3+p_5|4^-\rangle\right] - \\
&- \frac{\langle 1\,2\rangle}{\langle 1\,5\rangle\langle 2\,5\rangle s_{34}t_{125}}\left[\langle 2\,3\rangle[3\,4]\langle 3^-|p_2+p_5|1^-\rangle - \langle 3\,4\rangle[1\,4]\langle 2^-|p_1+p_5|4^-\rangle\right] - \\
&- \frac{[1\,4]}{s_{34}t_{134}s_{5H}}\langle 2^-|p_H|5^-\rangle\langle 3^-|p_1+p_4|5^-\rangle\Big\},
\end{aligned}
\tag{B.14}
$$

$$
\begin{aligned}
m_{5,3}&(1_q^+, 2_{\bar q}^-; 3_Q^-, 4_{\bar Q}^+; 5^+) = \\
= iA\Big\{ &- \frac{\langle 2\,3\rangle[1\,5]}{s_{34}t_{234}s_{5H}}\left[-[1\,5]\langle 1^-|p_2+p_3|4^-\rangle - [4\,5]t_{234}\right] - \\
&- \frac{1}{\langle 1\,5\rangle s_{34}t_{125}}\left[\langle 2\,3\rangle[3\,4]\langle 3^-|p_1+p_2|5^-\rangle + \langle 3\,4\rangle[4\,5]\langle 2^-|p_1+p_5|4^-\rangle\right] + \\
&+ \frac{\langle 1\,2\rangle}{\langle 1\,5\rangle\langle 2\,5\rangle s_{34}t_{125}}\left[\langle 2\,3\rangle[3\,4]\langle 3^-|p_2+p_5|1^-\rangle - \langle 3\,4\rangle[1\,4]\langle 2^-|p_1+p_5|4^-\rangle\right] + \\
&+ \frac{[1\,4]}{s_{34}t_{134}s_{5H}}\langle 2^-|p_H|5^-\rangle\langle 3^-|p_1+p_4|5^-\rangle\Big\},
\end{aligned}
\tag{B.15}
$$

$$
\begin{aligned}
m_{5,4}&(1_q^+, 2_{\bar q}^-; 3_Q^-, 4_{\bar Q}^+; 5^+) = \\
= iA\Big\{ &\frac{[1\,4]}{s_{12}t_{124}s_{5H}}\langle 2^-|p_1+p_4|5^-\rangle\langle 3^-|p_H|5^-\rangle - \\
&- \frac{\langle 1\,3\rangle}{\langle 1\,5\rangle\langle 3\,5\rangle s_{12}t_{345}}\left[-\langle 1\,2\rangle[1\,4]\langle 3^-|p_4+p_5|1^-\rangle + \langle 2\,3\rangle[1\,2]\langle 2^-|p_3+p_5|4^-\rangle\right] + \\
&+ \frac{1}{\langle 1\,5\rangle\langle 4\,5\rangle s_{12}t_{345}}[-\langle 2\,3\rangle^2[1\,2]\langle 1^-|p_4+p_5|3^-\rangle + \\
&\qquad + \langle 1\,2\rangle(s_{14}+s_{15})\langle 3^-|p_4+p_5|1^-\rangle - s_{12}s_{45}\langle 2\,3\rangle] - \\
&- \frac{\langle 2\,3\rangle[4\,5]}{s_{12}t_{123}s_{5H}}\left[-[1\,5]t_{123} - [4\,5]\langle 4^-|p_2+p_3|1^-\rangle\right]\Big\}.
\end{aligned}
\tag{B.16}
$$

All the other sub-amplitudes can be obtained by relabelling and by use of parity inversion, reflection symmetry and charge conjugation. For identical quarks, we must subtract from eq. (2.4) the same term with the antiquarks (but not the quarks) exchanged ($2_{\bar q} \leftrightarrow 4_{\overline{Q}}$).

B.4 Colour matrices for Higgs + five partons

Using eq. (2.1), the squared amplitude for Higgs + n gluons, summed over colours, is

$$\sum_{\text{colours}} |\mathcal{M}_n(1,\ldots,n)|^2 = 2^{n-4}(g^2)^{n-2} \sum_{i,j=1}^{(n-2)!} \tilde{c}_{ij}\ m_i\ (m_j)^*\,, \tag{B.17}$$

where the subscript i on m_i now refers to the sub-amplitude $m_n(1,\ldots,n)$ evaluated for the i^{th} permutation P_i in S_{n-2}. The colour matrix $\tilde{c}_{ij}$ is,

$$\tilde{c}_{ij} = \sum_{\text{colours}} (P_i\{F^{a_2}\cdots F^{a_{n-1}}\})_{a_1a_n} \left[(P_j\{F^{a_2}\cdots F^{a_{n-1}}\})_{a_1a_n}\right]^*, \tag{B.18}$$

where we have re-written the structure constants in terms of the SU(N_c) generators in the adjoint representation, $(F^a)_{bc} \equiv if^{bac}$. For $n = 5$ the colour matrix is,

$$\tilde{c}_{ij} = \frac{C_F N_c^4}{2} \begin{pmatrix} 4 & 2 & 2 & 1 & 1 & 0 \\ 2 & 4 & 1 & 0 & 2 & 1 \\ 2 & 1 & 4 & 2 & 0 & 1 \\ 1 & 0 & 2 & 4 & 1 & 2 \\ 1 & 2 & 0 & 1 & 4 & 2 \\ 0 & 1 & 1 & 2 & 2 & 4 \end{pmatrix}, \qquad m_i = \begin{pmatrix} (1,2,3,4,5) \\ (1,2,4,3,5) \\ (1,3,2,4,5) \\ (1,3,4,2,5) \\ (1,4,2,3,5) \\ (1,4,3,2,5) \end{pmatrix}. \tag{B.19}$$

Using eq. (2.3), the squared amplitude for Higgs + $(n-2)$ gluons and a $q\bar{q}$ pair, summed over colours, is

$$\sum_{\text{colours}} |\mathcal{M}_n(q,\bar{q};3,\ldots,n)|^2 = 2^{n-4}(g^2)^{n-2} \sum_{i,j=1}^{(n-2)!} d_{ij} m_i (m_j)^*\,, \tag{B.20}$$

where, as in eq. (B.17), the subscript i on m_i now refers to the sub-amplitude $m_n(1_q, 2_{\bar{q}}; 3,\ldots,n)$ evaluated for the i^{th} permutation P_i in S_{n-2}. The colour matrix d_{ij} is,

$$d_{ij} = \sum_{\text{colours}} (P_i\{T^{a_3}\ldots T^{a_n}\})_i^{\bar{\jmath}}\ \left[(P_j\{T^{a_3}\ldots T^{a_n}\})_i^{\bar{\jmath}}\right]^\dagger. \tag{B.21}$$

For $n = 5$ it is,

$$d_{ij} = C_F \begin{pmatrix} a & b & b & c & c & d \\ b & a & c & d & b & c \\ b & c & a & b & d & c \\ c & d & b & a & c & b \\ c & b & d & c & a & b \\ d & c & c & b & b & a \end{pmatrix}, \qquad m_i = \begin{pmatrix} (1,2;3,4,5) \\ (1,2;3,5,4) \\ (1,2;4,3,5) \\ (1,2;4,5,3) \\ (1,2;5,3,4) \\ (1,2;5,4,3) \end{pmatrix}, \tag{B.22}$$

where $a = N_c C_F^2$, $b = -C_F/2$, $c = 1/(4N_c)$ and $d = (N_c^2+1)/(4N_c)$.

Using eq. (B.8), the squared amplitude for Higgs + one gluon and two $q\bar{q}$ pairs, summed over colours, is

$$\sum_{\text{colours}} |\mathcal{M}_5(1_q, 2_{\bar{q}}; 3_Q, 4_{\overline{Q}}; 5)|^2 = \tag{B.23}$$
$$= \frac{g^6}{2} C_F \left[N_c^2(|m_{5,1}|^2 + |m_{5,2}|^2) + |m_{5,3}|^2 + |m_{5,4}|^2 + 2\,\mathrm{Re}(m_{5,1}^* + m_{5,2}^*)(m_{5,3} + m_{5,4})\right],$$

with $m_{5,i} = m_{5,i}(1_q, 2_{\bar{q}}; 3_Q, 4_{\overline{Q}}; 5)$ and $i = 1, \ldots, 4$. For identical quarks, we use eq. (B.8) and subtract the same term with the antiquarks (but not the quarks) exchanged ($2_{\bar{q}} \leftrightarrow 4_{\overline{Q}}$). Using the short-hand notations, $m_{5,i}(2,4) = m_{5,i}(1_q, 2_{\bar{q}}; 3_q, 4_{\bar{q}}; 5)$ and $m_{5,i}(4,2) = m_{5,i}(1_q, 4_{\bar{q}}; 3_q, 2_{\bar{q}}; 5)$, the squared amplitude summed over colours is

$$\sum_{\text{colours}} |\mathcal{M}_5(1_q, 2_{\bar{q}}; 3_q, 4_{\bar{q}}; 5)|^2 =$$
$$\begin{aligned} = \frac{g^6}{2} C_F \Bigg\{ & N_c^2 \sum_{i=1}^{2}(|m_{5,i}(2,4)|^2 + |m_{5,i}(4,2)|^2) + \sum_{i=3}^{4}(|m_{5,i}(2,4)|^2 + |m_{5,i}(4,2)|^2) - \\ & -2N_c\,\mathrm{Re}\Big[m_{5,1}^*(2,4)(m_{5,1}(4,2) + m_{5,2}(4,2) + m_{5,3}(4,2)) + m_{5,1}^*(4,2)m_{5,3}(2,4) + \\ & \quad + m_{5,2}^*(2,4)(m_{5,1}(4,2) + m_{5,2}(4,2) + m_{5,4}(4,2)) + m_{5,2}^*(4,2)m_{5,4}(2,4)\Big] + \\ & + 2\,\mathrm{Re}\Big[(m_{5,1}^*(2,4) + m_{5,2}^*(2,4))(m_{5,3}(2,4) + m_{5,4}(2,4)) + \\ & \quad + (m_{5,1}^*(4,2) + m_{5,2}^*(4,2))(m_{5,3}(4,2) + m_{5,4}(4,2))\Big] - \\ & - \frac{2}{N_c}\,\mathrm{Re}\left[(m_{5,3}^*(2,4) + m_{5,4}^*(2,4))(m_{5,3}(4,2) + m_{5,4}(4,2))\right] \Bigg\}. \end{aligned} \tag{B.24}$$

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